Name ____________________________

Student Number ___________________

Test 2
STA 347s 1991
Erindale College

1. (20 pts) The number of customers N entering a store on a given day is a discrete random variable with $E(N)=\mu_N$. The amount of money M_i spent by customer i is a continuous random variable with mean μ_M, and is independent of the number of customers who enter the store. Prove that the expected amount of money taken in on a given day is $\mu_N \mu_M$.

2. (20 pts) Let $\{X_0, X_1, X_2, \ldots\}$ be a stochastic process. For each of the ten definitions below, write in the letter of the term or phrase being defined. Write *only one letter in each space*; if more than one letter applies, write the **best** one.

a.	Ergodic	k.	Class
b.	i and j are disjoint	l.	Multipenetration
c.	State Space	m.	Irreducible
d.	Unitary	n.	i is accessible from j
e.	State d is Euphoric	o.	Strongly Consistent
f.	Random Walk	p.	Aperiodic
g.	j is accessible from i	q.	Recurrent
h.	Periodic with period d	r.	Markov Chain
i.	Transient	s.	i & j communicate
j.	Multivariate	t.	Reflexive

_____ Having period 1.

_____ P{starting in state i, the process will return to i} < 1

_____ $P_{ij}^{n}>0$ for some $n\geq 0$ and $P_{ji}^{m}>0$ for some $m\geq 0$.

_____ $P(X_{n+1}=j \mid X_n=i, X_{n-1}=i_{n-1}, \ldots, X_0=i_0)=P(X_{n+1}=j \mid X_n=i)$, for $n = 0, 1, \ldots$

_____ Having only one class

_____ P{starting in state i, the process will return to i} = 1

_____ $P_{i,i+1}=p$, $P_{i,i-1}=1-p$ for $i = 0, \pm 1, \pm 2, \ldots$

_____ The common support of $X_0, X_1, \ldots$

_____ $P_{ij}^{n}>0$ for some $n\geq 0$.

_____ A set of states that communicate with one another.

3. Here is the transition probability matrix for a Markov Chain.

$$P = \begin{array}{c} \\ \end{array}\begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \end{array}$$

$$P = \begin{bmatrix} 0.3 & 0.7 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.5 & 0.5 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.1 & 0.0 & 0.0 & 0.0 & 0.9 \\ 0.0 & 0.0 & 0.0 & 0.5 & 0.5 & 0.0 \\ 0.0 & 0.0 & 0.3 & 0.3 & 0.0 & 0.4 \\ 0.0 & 0.0 & 0.5 & 0.0 & 0.0 & 0.5 \end{bmatrix}$$

a) (10 pts) Identify the classes, and label each one as transient or recurrent.

b) (5 pts) If X_{14} is equally likely to be 0,1,2,3,4, or 5, what is $P(X_{15}=5)$?

c) (5 pts) What is $P_{1,4}^{100}$? (Think before you calculate).

4. Let $\{X_0, X_1, X_2, \ldots\}$ be a Markov Chain, and $\mathbf{P}^{(r)} = [(P^r_{ij})]$ be its matrix of r-step transition probabilities (r = 0, 1, ...).

a) (1 pt) What is the definition of $i \rightarrow j$ (give an inequality)

b) (1 pt) What is the definition of $j \rightarrow k$ (give an inequality)

c) (1 pt) What is the definition of $i \rightarrow k$ (give an inequality)

d) (2 pts) State the Chapman-Kolmogorov equations.

e) (15 pts) Use a-d to prove: $i \rightarrow j,\ j \rightarrow k \Rightarrow i \rightarrow k$. You cannot get any points if a-d are incorrect.

5. (20 pts) Suppose that the mood of an individual is a 3-state Markov chain with the following transition probability matrix:

$$\begin{bmatrix} 0.5 & 0 & 0.5 \\ 0.5 & 0 & 0.5 \\ 0 & 1 & 0 \end{bmatrix}.$$

What is the long-run proportion of time she is in each of the three states?

Total marks=100 points