Name Jenny

Student Number

STA 312 f2023 Quiz 1

Let the random variable X have an exponential distribution (see formula sheet on reverse), and let $Y = aX$, where the constant $a > 0$. Derive the probability density function of Y. Show your work. Do not forget to indicate where the density is non-zero.

For $y \geq 0$,

$$f_Y(y) = \frac{d}{dy} F_Y(y) = \frac{d}{dy} P(Y \leq y) = \frac{d}{dy} P(aX \leq y)$$

$$= \frac{d}{dy} P(X \leq \frac{1}{a} y) = \frac{d}{dy} F_X(\frac{1}{a} y)$$

$$= f_X(\frac{1}{a} y) \cdot \frac{1}{a} = \frac{\lambda}{a} e^{-\frac{\lambda}{a} y}, \text{ so}$$

$$f_Y(y) = \begin{cases} \frac{\lambda}{a} e^{-\frac{\lambda}{a} y} & \text{for } y \geq 0 \\ 0 & \text{for } y < 0 \end{cases}$$

Maybe 3 marks for the support.