

Sample Questions: Censoring and Likelihood

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The Pareto density is $f(t|\theta) = \begin{cases} \frac{\theta}{t^{\theta+1}} & \text{for } t \geq 1 \\ 0 & \text{for } t < 1 \end{cases}$.

1. Let $(T_1, \delta_1), \dots, (T_n, \delta_n)$ be a random sample from a Pareto distribution with right censoring. Find the maximum likelihood estimate of θ .

$$S(t) = \int_t^{\infty} \theta x^{-\theta-1} dx = \theta \frac{x^{-\theta-1+1}}{-\theta} \Big|_t^{\infty}$$

$$= -\frac{1}{x^{\theta}} \Big|_t^{\infty} = -\left(0 - \frac{1}{t^{\theta}}\right) = \frac{1}{t^{\theta}}$$

$$l(\theta) = \sum_{i=1}^n \delta_i (\log \theta - (\theta+1) \log t_i) + \sum_{i=1}^n (1-\delta_i) (-\theta) \log t_i$$

$$= \sum_{i=1}^n \delta_i \log \theta - \theta \sum_{i=1}^n \delta_i \log t_i - \sum_{i=1}^n \delta_i \log t_i - \theta \sum_{i=1}^n \log t_i + \theta \sum_{i=1}^n \delta_i \log t_i$$

$$l'(\theta) = \frac{\sum_{i=1}^n \delta_i}{\theta} - 0 - \sum_{i=1}^n \log t_i \stackrel{\text{set}}{=} 0$$

$$\Rightarrow \frac{\sum_{i=1}^n \delta_i}{\theta} = \sum_{i=1}^n \log t_i \Rightarrow \hat{\theta} = \frac{\sum_{i=1}^n \delta_i}{\sum_{i=1}^n \log t_i}$$

$$l''(\theta) = \frac{d}{d\theta} \left(\theta^{-1} \sum_{i=1}^n \delta_i - \sum_{i=1}^n \log t_i \right) = -\frac{\sum \delta_i}{\theta^2} < 0 \cap \text{MAX}$$

2. Find the standard error of $\hat{\theta}$.

$$\hat{\sigma}_n = \frac{-1}{l''(\hat{\theta})}$$

$$= \frac{\hat{\theta}^2}{\sum_{i=1}^n \delta_i}$$

$$SE = \frac{\hat{\theta}}{\sqrt{\sum_{i=1}^n \delta_i}}$$

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<http://www.utstat.toronto.edu/brunner/oldclass/312f23>