

# Poisson Regression

STA 312 Fall 2022

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# Regression: Outcomes are Counts

- Poisson process model roughly applies
- Examples: Relationship of explanatory variables to
  - Number of children
  - Number of typos in a short document
  - Number of workplace accidents in a short time period
  - Number of marriages
- For large  $\lambda$ , CLT says a normality assumption is okay, but not constant variance

# Linear Model for $\log \lambda$

- $\log \lambda = \beta_0 + \beta_1 x_1 + \dots + \beta_{p-1} x_{p-1}$
- Implicitly for  $i = 1, \dots, n$
- Everybody in the sample has a different  $\lambda = \lambda_i$
- Take exponential function of both sides
- Substitute into Poisson likelihood
- Maximum likelihood as usual
- Likelihood ratio tests, etc.

$$\log \lambda = \beta_0 + \beta_1 x_1 + \dots + \beta_{p-1} x_{p-1}$$

- Increase  $x_k$  with everything else held constant, and
  - $\log \lambda$  increases by  $\beta_k$
  - $\lambda$  is multiplied by  $e^{\beta_k}$

# Back to the job study: n=200 Students

- 106 employed in a job related to field of study
- 74 employed in a job unrelated to field of study
- 20 unemployed
- Could be independent Poisson processes
  
- Conditionally on the total number of students, multinomial with
  - $\pi_1 = \lambda_1 / (\lambda_1 + \lambda_2 + \lambda_3)$
  - $\pi_2 = \lambda_2 / (\lambda_1 + \lambda_2 + \lambda_3)$
  - $\pi_3 = \lambda_3 / (\lambda_1 + \lambda_2 + \lambda_3)$

# Poisson regression with dummy variables

<b>Job Status</b>	$d_1$	$d_2$	$\log \lambda = \beta_0 + \beta_1 d_1 + \beta_2 d_2$
Related	0	0	$\beta_0 = \log \lambda_1$
Unrelated	1	0	$\beta_0 + \beta_1 = \log \lambda_2$
Unemployed	0	1	$\beta_0 + \beta_2 = \log \lambda_3$

On average, we expect  $e^{\beta_2}$  times as many unemployed students as students with jobs related to their fields of study.

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