

Interactions in logistic Regression

①

Quant by Quant and Quant by Cat
Put them aside.

Categorical by Categorical

Berkeley Grad admissions data

	A	B	C	D	E	F
F	β_{11}	β_{12}	β_{13}	β_{14}	β_{15}	β_{16}
M	β_{21}	β_{22}	β_{23}	β_{24}	β_{25}	β_{26}

Cell means model

$$\log \frac{\pi}{1-\pi} = \beta_{11} x_{11} + \beta_{12} x_{12} + \dots + \beta_{26} x_{26}$$

↑
0-1 indicator

Interaction would look like

(2)

$$H_0: \beta_{11} - \beta_{21} = \beta_{12} - \beta_{22} = \beta_{13} - \beta_{23} = \dots = \beta_{16} - \beta_{26}$$

Each term is a difference in log odds

In terms of odds,

$$e^{\beta_{11} - \beta_{21}} = \frac{e^{\beta_{11}}}{e^{\beta_{21}}} \text{ Odds ratio}$$

So H_0 says all 6 odds ratios are the same

Can (should) look at which odds ratios are different from which others

$$\binom{6}{2} = 15 \text{ pairwise comparisons}$$

Use effect coding first

(5)

	A	B	C	D	E	F	
F							$\beta_0 + \beta_1$
M							$\beta_0 - \beta_1$
	$\beta_0 + \beta_2$	$\beta_0 + \beta_3$	$\beta_0 + \beta_4$	$\beta_0 + \beta_5$	$\beta_0 + \beta_6$		β_0

$$\log \frac{\pi}{1-\pi} = \beta_0 + \beta_1 g + \beta_2 d_1 + \beta_3 d_2 + \dots + \beta_{11} g d_5$$

$$\beta_0 - \beta_2 - \beta_3 - \beta_4 - \beta_5 - \beta_6$$

How many interaction terms

$$(6-1)(2-1) = 5$$