

Likelihood function: $\theta = (\beta, \sigma^2)$

Denoting $\underset{\sim}{y}_i = \begin{pmatrix} y_{i1} \\ y_{i2} \\ \vdots \\ y_{im} \end{pmatrix}$

$$L(\theta) = \prod_{i=1}^n P_{\theta} \{ \underset{\sim}{Y}_i = \underset{\sim}{y}_i \}$$

$$= \prod_{i=1}^n \int_{-\infty}^{\infty} P_{\theta} \{ \underset{\sim}{Y}_i = \underset{\sim}{y}_i \mid B_i = b_i \} f(b_i) db_i$$

$$= \prod_{i=1}^n \int_{-\infty}^{\infty} \prod_{j=1}^m \left(\frac{e^{x_{ij}^T \beta + b_i}}{1 + e^{x_{ij}^T \beta + b_i}} \right)^{y_{ij}} f(b_i \mid \sigma^2) db_i$$