Randomized Block Designs[1]

STA305 Winter 2014

[1] See last slide for copyright information.

Background Reading

Optional

- Photocopy 2 from an old textbook; see course website. It's only four pages.
- The Wikipedia has a page that's okay, but it's not as clear as the old textbook.

Goal of Blocking

- Goal is increased precision, like the analysis of covariance.
- In ANCOVA, random assignment to treatments ensures that covariates are statistically independent of the experimental treatment.
- But there could still be some relationship between treatment and covariates in the sample, just by chance.
- Example: More older people could wind up in the placebo group just by luck.
- Blocking aims to reduce this source of noise through the *design*.

Basic idea

- Blocking variables, like covariates, are nuisance variables that are known to be strongly associated with the response.
- For example
  - Some parts of a field are just more fertile; crops *always* grow better there.
  - Some waiters always get more tips.
  - Some high schools always get higher test scores.

- So randomly assign experimental units to treatments *within blocks*.
- One full set of treatments for each block is called a "complete block design."

Examples

Randomly assign experimental units to treatments within blocks.

- Compare two contact lens cleaning solutions. Block is the person. For each person, randomly assign one eye to each treatment.
- Compare p crop fertilizers. Divide available land into *blocks*, subdivide each block into p plots, and randomly assign plots to fertilizers within each block.
- Compare three programs for training kindergarten teachers in music. For a set of schools with at least three kindergarten classrooms, randomly choose three if necessary, and then randomly assign one of the three teachers to each training program, within each school.

More examples

- Compare four off-season training programs for high school basketball teams. Sort the teams according to won-lost record last season, and then divide into blocks of four teams with similar (though not identical) records. Randomly assign teams to programs, within each block.
- Assess the effect of chocolate cake recipe on amount of tip at a restaurant. The waiter recommends the cake and says there's a special low price (a lie). For each waiter separately, tables are randomly assigned to one of three recipes, in blocks of $3! = 6$ consecutive tables.
- In the last example, there were two blocking variables, waiter and order. There could be multiple blocks of six for each waiter: a "generalized" block design.

Counting problems

Suppose there are p treatments, arranged in a complete randomized block design with k blocks. This means each treatment appears exactly once within each block.

- How many experimental units are required?
- In how many ways can the units within each block be assigned to experimental treatments?
- In how many total ways can the experimental units be assigned to treatments?
- What is the maximum number of values in the permutation distribution of the test statistic?

Statistical model

F-tests are approximations of the permutation tests

Ordinary factorial ANOVA (regression) model, but with *no interactions between blocks and treatments*.

- Consider p treatments arranged in a complete randomized block design with k blocks, so that each treatment appears exactly once within each block.
- $n = pk$, and total $df = pk - 1$.
- Fill in the degrees of freedom for an ANOVA summary table.
- Make rows for Blocks, Treatments, Blocks $\times$ Treatments, Error, and Total$= pk - 1$. Fill in the other numbers.
- This is why there is no interaction between blocks and treatments.

Generalized randomized block design

- More than one (full) set of experimental treatments in each block.
- Block is just another factor, though it's observed rather than manipulated.
- Interactions are testable.

Latin Square designs

Two blocking variables

A	B	C	D
B	A	D	C
C	D	A	B
D	C	B	A

There can be both blocking and covariates in the same experiment

For example,

- Milk cows are randomly assigned to different types of feed. Response variable is volume of milk produced.
- Age of cow is the blocking variable: Important.
- Weight is important too, but it's hard to block on weight and age at the same time.
- Make weight a covariate.

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