This is a preview of what students will see when they are submitting the assignment. Interactive features are disabled.

Quiz 3

Due: Thursday October 1, 2020 6:40 PM (EDT)

Submit your assignment

Help

After you have completed the assignment, please save, scan, or take photos of your work and upload your files to the questions below. Crowdmark accepts PDF, JPG, and PNG file formats.

Q1 (3 points)

Let the matrix $\mathbf{A} = \begin{pmatrix} 4 & 0 & -2 \\ 0 & 5 & 1 \\ -2 & 1 & 3 \end{pmatrix}$. Using R, calculate the eigenvalues of $\mathbf{A}^{-1}$.

Capture an image of your *complete* R input and output. Circle or highlight your answer. It does not matter how you capture the image. Even a photo of your computer screen is okay if it's legible.

Q2 (3 points)

Let the $p \times p$ matrix $\mathbf{A}$ be positive semi-definite, meaning $\mathbf{v}'\mathbf{A}\mathbf{v} \geq 0$ for any $p \times 1$ vector $\mathbf{v}$. Let $\mathbf{B}$ be another $p \times p$ matrix. Show that $\mathbf{B}'\mathbf{A}\mathbf{B}$ is also positive semi-definite.

Q3 (4 points)

Let the random vector $\mathbf{x}$ have expected value $\boldsymbol{\mu}$ and variance-covariance matrix $\boldsymbol{\Sigma}$. Let $\mathbf{y} = [y_j] = \mathbf{C}'\mathbf{x}$, where the columns of $\mathbf{C}$ are the eigenvectors of $\boldsymbol{\Sigma}$. Prove $Cov(y_i, y_j) = 0$ for $i \neq j$.

Quiz 3

(1)

```
>
>
>
>
>
>
> A = rbind(c(4, 0, -2),
+           c(0, 5,  1),
+           c(-2,1,  3))
> # There are two good ways to do it.
> eigen(solve(A))$values
[1] 0.7886751 0.2113249 0.1666667
> 1/eigen(A)$values
[1] 0.1666667 0.2113249 0.7886751
>
```

MUST write this

(2) $v'B'ABv = (\underbrace{Bv}_{p\times 1})' A \underbrace{Bv}_{p\times 1} = x'Ax \geq 0$

because A is positive semi-definite.

(3) $\text{cov}(y) = \text{cov}(C'x) = C'\text{cov}(x)C = C'\Sigma C$

$= \underbrace{C'C}_{I} D \underbrace{C'C}_{I} = D$, a diagonal matrix,

and the result follows (because all the off-diagonal elements of this covariance matrix are zero).