Quiz 11

Due: Thursday December 3, 2020 6:40 PM (EST)

Submit your assignment

Help

After you have completed the assignment, please save, scan, or take photos of your work and upload your files to the questions below. Crowdmark accepts PDF, JPG, and PNG file formats.

Q1 (5 points)

Independently for $i = 1, \ldots, n$, let $y_i = \beta x_i + \epsilon_i$, where $\epsilon_i \sim N(0, \sigma^2 x_i)$. The ordinary least squares estimate of β is $\frac{\sum_{i=1}^n x_i y_i}{\sum_{i=1}^n x_i^2}$, but that's not optimal because of the unequal variances. Using scalar calculations only (no matrices), show that the weighted least squares estimate of β is $\frac{\bar{y}}{\bar{x}}$. The formula for the ordinary least squares estimate should be useful.

Q2 (~~10~~ 5 points)

The file http://www.utstat.toronto.edu/~brunner/data/legal/xy.data.txt has a generic set of x and y data. It's not quite right for Problem 1, because there are some $x = 0$ values. To fix it up, add one to all the x values. Then, using the `weights` option in R's `lm` function, verify that the weighted least squares estimate really is $\frac{\bar{y}}{\bar{x}}$. **Circle the estimate**. Upload your complete R input and output.

Quiz 11

(1)

$$y_i = \beta x_i + \varepsilon_i \iff \frac{1}{\sqrt{x_i}}$$

$$\iff \frac{1}{\sqrt{x_i}} y_i = \frac{1}{\sqrt{x_i}} \beta x_i + \frac{1}{\sqrt{x_i}} \varepsilon_i$$

$$= \beta \sqrt{x_i} + \frac{1}{\sqrt{x_i}} \varepsilon_i$$

$$\Longrightarrow y_i^* = \beta x_i^* + \varepsilon_i^*, \text{ & the WLS estimate is}$$

$$\hat{\beta}_{WLS} = \frac{\sum_{i=1}^{n} x_i^* y_i^*}{\sum_{i=1}^{n} x_i^{*2}} = \frac{\sum_{i=1}^{n} \sqrt{x_i} \frac{1}{\sqrt{x_i}} y_i}{\sum_{i=1}^{n} (\sqrt{x_i})^2}$$

$$= \frac{\sum_{i=1}^{n} y_i}{\sum_{i=1}^{n} x_i} \cdot \frac{\frac{1}{n}}{\frac{1}{n}} = \frac{\bar{y}}{\bar{x}}$$

R input and output for Quiz 11

```
> xy = read.table('http://www.utstat.toronto.edu/~brunner/data/legal/xy.data.txt')
> attach(xy)
> x = x+1
> summary(lm(y ~ 0 + x, weights = 1/x))

Call:
lm(formula = y ~ 0 + x, weights = 1/x)

Weighted Residuals:
     Min       1Q   Median       3Q      Max
-14.1682  -3.7483  -0.0829   3.8404   9.7536

Coefficients:
  Estimate Std. Error t value Pr(>|t|)
x   1.4082     0.1812   7.771 4.07e-13 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 5.074 on 199 degrees of freedom
Multiple R-squared:  0.2328,  Adjusted R-squared:  0.229
F-statistic: 60.39 on 1 and 199 DF,  p-value: 4.068e-13

> mean(y)/mean(x)
[1] 1.408176
```