Assignment 8

(1) (a) $y_i = \beta_0 + \beta_1 x_{i,1} + \beta_2 x_{i,2} + \beta_3 x_{i,3} + \varepsilon_i$

(b) $E(y) = \beta_0 + 70\beta_1 + 85\beta_2 + 65\beta_3$

(c) β_3

(d) i. $H_0: \beta_1 = \beta_2 = \beta_3 = 0$

ii. $H_0: C\beta = t$

$$\underbrace{\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}}_{C} \underbrace{\begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix}}_{\beta} = \underbrace{\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}}_{t}$$

iii) $y_i = \beta_0 + \varepsilon_i$

(e) i. $H_0: \beta_1 = 0$

ii. $$\underbrace{(0\,1\,0\,0)}_{C} \underbrace{\begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix}}_{\beta} = \underbrace{(0)}_{t}$$

iii. $y_i = \beta_0 + \beta_2 x_{i2} + \beta_3 x_{i3} + \varepsilon_i$

(f) i. $H_0: \beta_3 = 0$

ii. $$\underbrace{(0\,0\,0\,1)}_{C} \underbrace{\begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix}}_{\beta} = \underbrace{(0)}_{t}$$

iii. $y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \varepsilon_i$

(1g) i. $H_0: \beta_2 = 0$

ii. $\underbrace{(0\ 0\ 1\ 0)}_{C} \underbrace{\begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix}}_{\beta} = \underbrace{(0)}_{A}$

iii. $y_i = \beta_0 + \beta_1 x_{i1} + \beta_3 x_{i3} + \varepsilon_i$

(h) i. $H_0: \beta_1 = \beta_3 = 0$

ii. $\underbrace{\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}}_{C} \underbrace{\begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix}}_{\beta} = \underbrace{\begin{pmatrix} 0 \\ 0 \end{pmatrix}}_{A}$

iii. $y_i = \beta_0 + \beta_2 x_{i,2} + \varepsilon_i$

(i) i. $H_0: \beta_0 = 0,\ \beta_1 = 0.4,\ \beta_2 = 0.2,\ \beta_3 = 0.4$

ii. $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0.4 \\ 0.2 \\ 0.4 \end{pmatrix}$

iii. $y_i = 0.4 x_{i1} + 0.2 x_{i2} + 0.4 x_{i3} + \varepsilon_i$

(2) (a) i. 86.5

ii. $r = 0.0331$

iii. No.

First note that $\frac{1}{n}\sum_{i=1}^{n} 10x_i = 10\bar{x}$, and

$$r = \frac{\sum_{i=1}^{n}(10x_i - 10\bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}(10x_i - 10\bar{x})^2 \sum_{i=1}^{n}(y_i - \bar{y})^2}} = \frac{\cancel{10}\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}\cancel{100}(x_i - \bar{x})^2 \sum_{i=1}^{n}(y_i - \bar{y})^2}}$$

= formula for r on the formula sheet.

iv. $0.396^2 = 0.1568$

(b) Okay...

(c) $\hat{\beta}_2 = -0.2934$

(d) $n = 58$

(e) $k = 3$

(f) X is 58×4

(g) $\hat{\beta}$ is 4×1

(h) $\hat{\varepsilon}$ is 58×1

(i) $\hat{\varepsilon}'\hat{\varepsilon}$ is 1×1

(j) $\hat{y}$ is 58×1

(k) H is 58×58

(l) From summary, $14.54 = \sqrt{\hat{\varepsilon}'\hat{\varepsilon}/(58-3-1)}$

$\Rightarrow 14.54^2(54) = 11416.23$

More accurate is to use the residuals function, getting 11410.04

(2m) $R^2 = \frac{SSR}{SST} = \frac{SST - SSE}{SST} = 1 - \frac{SSE}{SST}$

$\Rightarrow \frac{SSE}{SST} = 1 - R^2 \Rightarrow SST = \frac{SSE}{1-R^2}$

From (ℓ), $SSE = 11416.23$, & $R^2 = 0.2662$, so

$SST = \frac{11416.23}{1 - 0.2662} = 15557.69$

Easier & more accurate is

$s^2 = \frac{\Sigma(y_i - \bar{y})^2}{n-1} = \frac{SST}{n-1}$ so

$SST = s^2(n-1) = 15548.34$ Less rounding error.

(n) $\hat{y} = 46.38757 \approx 46.39$

(o) $\hat{\beta}_3 = 0.325$

(p) 42,218

(q)

H_0	Test statistic	p-value	Reject H_0?
$\beta_1 = \beta_2 = \beta_3 = 0$	$F = 6.528$	0.00075	Yes
$\beta_0 = 0$	$t = 0.560$	0.57746	No
$\beta_1 = 0$	$t = 2.743$	0.00825	Yes
$\beta_2 = 0$	$t = -1.538$	0.12977	No
$\beta_3 = 0$	$t = 2.343$	0.02283	Yes

(2r) $R^2 = 0.2662$

(s) i. $H_0: \beta_1 = 0$

ii. $t = 2.743$, $df = 54$, $p = 0.00825$

iii. Yes

iv. Yes

v. Controlling for computer average and midterm, the higher a student's quiz average, the better he or she tended to do on the final exam.

vi. Yes. $F = 7.52173894$, $t^2 = 2.743^2 = 7.524049$
Rounding error.

vii. Got $F = 7.5217$

viii. $p = 0.1222615$ or $p = 0.12$ is ~~good~~ enough.

(t) 95% CI (0.158, 1.016) does not contain $\beta_1 = 0$.

(u) i. $\hat{y} = 49.44828 = \bar{y}$

ii. Assignment 4, Question 3.

iii. (45.62161, 53.27495)

iv. (45.10562, 53.79093)

Different model: Estimate of σ^2 is a little smaller, so confidence interval is narrower.

(2v) i. Controlling for quiz average and computer average, students with higher marks on the midterm tended to do better on the final exam.

ii. Allowing for mark on the midterm test and quiz average, there is no evidence that computer average is a useful predictor of score on the final exam.

iii. Taking into account mark on the midterm test and computer average, students with higher quiz averages tended to do better on the final exam.

(vi) $0.00825 * 3 = 0.025 < 0.05$ $\ngeq$

$0.02283 * 3 = 0.068 > 0.05$

So the only conclusion is 2v part iii above.

(2x) i) $H_0: \beta_1 = \beta_2 = 0$

ii)
$$\underbrace{\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}}_{C} \underbrace{\begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix}}_{\beta} = \underbrace{\begin{pmatrix} 0 \\ 0 \end{pmatrix}}_{t}$$

iii) $y_i = \beta_0 + \beta_3 x_{i,3} + \varepsilon_i$

iv) $F^* = 3.\overset{79}{\cancel{97}}8$

v) Same F^*

vi) $p = 0.02862$

vii) Yes

viii) Yes

ix) $p = \overset{0.1233}{\cancel{0.0657}}$ (Different p, of course)

x) Controlling for mark on the midterm test, either quiz average, or computer average is related to mark on the final exam — or possibly both.