STA 302 Formulas

$M_y(t) = E(e^{yt})$

$M_{ay}(t) = M_y(at)$

$M_{y+a}(t) = e^{at}M_y(t)$

$M_{\sum_{i=1}^n y_i}(t) = \prod_{i=1}^n M_{y_i}(t)$

$y \sim N(\mu, \sigma^2)$ means $M_{_y}(t) = e^{\mu t + \frac{1}{2}\sigma^2 t^2}$

$y \sim \chi^2(\nu)$ means $M_{_y}(t) = (1-2t)^{-\nu/2}$

If $w = w_1 + w_2$ with w_1 and w_2 independent, $w \sim \chi^2(\nu_1 + \nu_2)$, $w_2 \sim \chi^2(\nu_2)$ then $w_1 \sim \chi^2(\nu_1)$

$y_i = \beta_0 + \beta_1 x_i + \epsilon_i$

$b_0 = \overline{y} - b_1\overline{x}$

$b_1 = \frac{\sum_{i=1}^n (x_i-\overline{x})(y_i-\overline{y})}{\sum_{i=1}^n (x_i-\overline{x})^2} = \frac{\sum_{i=1}^n x_i y_i - n\,\overline{x}\,\overline{y}}{\sum_{i=1}^n x_i^2 - n\overline{x}^2}$

$r = \frac{\sum_{i=1}^n (x_i-\overline{x})(y_i-\overline{y})}{\sqrt{\sum_{i=1}^n (x_i-\overline{x})^2}\sqrt{\sum_{i=1}^n (y_i-\overline{y})^2}}$

Columns of A *linearly dependent* means there is a vector $\mathbf{v} \neq \mathbf{0}$ with $A\mathbf{v} = \mathbf{0}$.

Columns of A *linearly independent* means that $A\mathbf{v} = \mathbf{0}$ implies $\mathbf{v} = \mathbf{0}$.

Matrix A is non-negative definite means $\mathbf{v}'A\mathbf{v} \geq 0$.

Matrix A is positive definite means $\mathbf{v}'A\mathbf{v} > 0$ if $\mathbf{v} \neq \mathbf{0}$.

$\Sigma = CDC'$

$\Sigma^{-1} = CD^{-1}C'$

$\Sigma^{1/2} = CD^{1/2}\mathbf{C}'$

$\Sigma^{-1/2} = CD^{-1/2}C'$

$cov(\mathbf{y}) = E\left\{(\mathbf{y}-\boldsymbol{\mu}_y)(\mathbf{y}-\boldsymbol{\mu}_y)'\right\}$

$cov(\mathbf{y},\mathbf{t}) = E\left\{(\mathbf{y}-\boldsymbol{\mu}_y)(\mathbf{t}-\boldsymbol{\mu}_t)'\right\}$

$cov(A\mathbf{y}) = A\,cov(\mathbf{y})A'$

$cov(A\mathbf{y}, B\mathbf{y}) = A\,cov(\mathbf{y})B'$

$M_{\mathbf{y}}(\mathbf{t}) = E(e^{\mathbf{t}'\mathbf{y}})$

$M_{A\mathbf{y}}(\mathbf{t}) = M_{\mathbf{y}}(A'\mathbf{t})$

$M_{\mathbf{y}+\mathbf{c}}(\mathbf{t}) = e^{\mathbf{t}'\mathbf{c}}M_{\mathbf{y}}(\mathbf{t})$

$\mathbf{y} \sim N_p(\boldsymbol{\mu}, \Sigma)$ means $M_{\mathbf{y}}(\mathbf{t}) = e^{\mathbf{t}'\boldsymbol{\mu} + \frac{1}{2}\mathbf{t}'\Sigma\mathbf{t}}$

$\mathbf{y}_1$ and $\mathbf{y}_2$ are independent if and only if $M_{(\mathbf{y}_1,\mathbf{y}_2)}(\mathbf{t}_1,\mathbf{t}_2) = M_{\mathbf{y}_1}(\mathbf{t}_1)M_{\mathbf{y}_2}(\mathbf{t}_2)$

If $\mathbf{y} \sim N_p(\boldsymbol{\mu}, \Sigma)$, then $A\mathbf{y} + \mathbf{c} \sim N_q(A\boldsymbol{\mu} + \mathbf{c}, A\Sigma A')$, and $w = (\mathbf{y}-\boldsymbol{\mu})'\Sigma^{-1}(\mathbf{y}-\boldsymbol{\mu}) \sim \chi^2(p)$

$y_i = \beta_0 + \beta_1 x_{i1} + \cdots + \beta_k x_{ik} + \epsilon_i$

$\epsilon_1, \ldots, \epsilon_n$ independent $N(0, \sigma^2)$

$\mathbf{y} = X\boldsymbol{\beta} + \boldsymbol{\epsilon}$ with $\boldsymbol{\epsilon} \sim N(\mathbf{0}, \sigma^2 I_n)$

$\mathbf{b} = (X'X)^{-1}X'\mathbf{y} \sim N_{k+1}\left(\boldsymbol{\beta}, \sigma^2(X'X)^{-1}\right)$

$\widehat{\mathbf{y}} = X\mathbf{b} = H\mathbf{y}$, where $H = X(X'X)^{-1}X'$

$\mathbf{e} = \mathbf{y} - \widehat{\mathbf{y}} = (I-H)\mathbf{y}$

$\mathbf{b}$ and $\mathbf{e}$ are independent under normality.

$\frac{SSE}{\sigma^2} = \frac{\mathbf{e}'\mathbf{e}}{\sigma^2} \sim \chi^2(n-k-1)$

$\sum_{i=1}^n (y_i-\overline{y})^2 = \sum_{i=1}^n (y_i-\widehat{y}_i)^2 + \sum_{i=1}^n (\widehat{y}_i-\overline{y})^2$

$SST = SSE + SSR$ and $R^2 = \frac{SSR}{SST}$

$t = \frac{Z}{\sqrt{W/\nu}} \sim t(\nu)$

$F = \frac{W_1/\nu_1}{W_2/\nu_2} \sim F(\nu_1, \nu_2)$

$t = \frac{\boldsymbol{\ell}'\mathbf{b} - \boldsymbol{\ell}'\boldsymbol{\beta}}{s\sqrt{\boldsymbol{\ell}'(X'X)^{-1}\boldsymbol{\ell}}} \sim t(n-k-1)$

$\boldsymbol{\ell}'\mathbf{b} \pm t_{\alpha/2}\, s\sqrt{\boldsymbol{\ell}'(X'X)^{-1}\boldsymbol{\ell}}$

$F^* = \frac{(\mathbf{Cb}-\boldsymbol{\gamma})'(\mathbf{C}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{C}')^{-1}(\mathbf{Cb}-\boldsymbol{\gamma})}{m\,s^2} \stackrel{H_0}{\sim} F(m, n-k-1)$

$F^* = \frac{SSR_F - SSR_R}{m\,s^2} = \left(\frac{n-k-1}{m}\right)\left(\frac{a}{1-a}\right)$

$s^2 = \frac{SSE}{n-k-1} = \frac{\mathbf{e}'\mathbf{e}}{n-k-1}$

$a = \frac{R_F^2 - R_R^2}{1-R_R^2} = \frac{mF}{n-k-1+mF}$

$t = \frac{y_0 - \mathbf{x}_0'\mathbf{b}}{s\sqrt{1+\mathbf{x}_0'(X'X)^{-1}\mathbf{x}_0}} \sim t(n-k-1)$

$\mathbf{x}_0'\mathbf{b} \pm t_{\alpha/2}\, s\sqrt{1+\mathbf{x}_0'(X'X)^{-1}\mathbf{x}_0}$
