# Assignment 7

① You cannot prove it. The claim is false, meaning not true in general. All you need is one scatterplot where the points are not colinear to see this.

② Computer, except (r2)

Have $s, n, k, R^2$

$SSE = s^2(n-k-1)$, and

$$R^2 = \frac{SSR}{SST} = \frac{SST - SSE}{SST} = 1 - \frac{SSE}{SST}$$

$$\Longrightarrow \frac{SSE}{SST} = 1 - R^2 \Longrightarrow SST = \frac{SSE}{1-R^2}$$

③ (a) $E(y|x) = \beta_1 r_1 + \beta_2 r_2 + \beta_3 r_3 + \beta_4 r_4 + \beta_5 x$

$x$ ← Income

(b)

| | $r_1$ | $r_2$ | $r_3$ | $r_4$ | $E(y\|x)$ |
|---|---|---|---|---|---|
| NE | 1 | 0 | 0 | 0 | $\beta_1 + \beta_5 x$ |
| NC | 0 | 1 | 0 | 0 | $\beta_2 + \beta_5 x$ |
| S | 0 | 0 | 1 | 0 | $\beta_3 + \beta_5 x$ |
| W | 0 | 0 | 0 | 1 | $\beta_4 + \beta_5 x$ |

(3c) (i) $H_0: \beta_1 = \beta_2 = \beta_3 = \beta_4$

(ii) $H_0: \beta_1 = \beta_2$

(iii) $H_0: \beta_1 = \beta_4$

(iv) $H_0: \beta_3 = \frac{1}{3}(\beta_1 + \beta_2 + \beta_4)$

(v) $H_0: \beta_1 + \beta_2 = \beta_3 + \beta_4$

(vi) $H_0: \beta_5 = 0$

(4a) $E(y|x) = \beta_0 + \beta_1 r_1 + \beta_2 r_2 + \beta_3 r_3 + \beta_4 x$

(b)

| | $r_1$ | $r_2$ | $r_3$ | $E(y\|x)$ |
|---|---|---|---|---|
| NE | 1 | 0 | 0 | $\beta_0 + \beta_1 + \beta_4 x$ |
| NC | 0 | 0 | 0 | $\beta_0 + \beta_4 x$ |
| S | 0 | 1 | 0 | $\beta_0 + \beta_2 + \beta_4 x$ |
| W | 0 | 0 | 1 | $\beta_0 + \beta_3 + \beta_4 x$ |

(c) (i) $H_0: \beta_1 = \beta_2 = \beta_3 = 0$

(ii) $H_0: \beta_1 = 0$

(iii) $H_0: \beta_1 = \beta_3$

(iv) $H_0: \beta_0 + \beta_2 = \frac{1}{3}(\beta_0 + \beta_1 + \beta_0 + \beta_0 + \beta_3)$

$\iff \cancel{\beta_0} + \beta_2 = \cancel{\beta_0} + \beta_1/3 + \beta_3/3$

$\iff 3\beta_2 = \beta_1 + \beta_3$

(v) $H_0: \cancel{\beta_0} + \beta_1 + \cancel{\beta_0} = \cancel{\beta_0} + \beta_2 + \cancel{\beta_0} + \beta_3 \iff \beta_1 = \beta_2 + \beta_3$

(vi) $H_0: \beta_4 = 0$

(5) $\sum_{i=1}^{n} e_i = 1'e = (X\nu)'e = \nu' \underbrace{X'e}_{} = \nu' 0 = 0_{1\times 1}$

(6)
$$A = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$
(Note multiplication by the identity matrix)

(7) (a) $a = (W'W)^{-1}W'y = \left((XA)'XA\right)^{-1}(XA)'y$

$= (A'X'XA)^{-1}A'X'y$

$= A^{-1}(X'X)^{-1}\underbrace{A'^{-1}A'}_{}X'y$

$= A^{-1}(X'X)^{-1}X'y = A^{-1}b$ (Note parallel to $\alpha = A^{-1}\beta$)

(b) $\hat{y} = Wa = XAA^{-1}b = Xb$, same $\hat{y}$

(c) $H_0: C\beta = \gamma \iff CAA^{-1}\beta = \gamma$

$\iff CA\alpha = \gamma$

$\iff H_0: C_2\alpha = \gamma$, where $C_2 = CA$

(d) For the transformed model, testing $H_0: C_2\alpha = \gamma$,

$F^* = (C_2a-\gamma)'(C_2(W'W)^{-1}C_2')^{-1}(C_2a-\gamma)/(ms^2)$

$= \dfrac{(C\underbrace{AA^{-1}}_{I}b-\gamma)'(\underbrace{CAA^{-1}(X'X)^{-1}A'^{-1}}_{\text{From part (a)}}(CA)')^{-1}(Cb-\gamma)}{(ms^2)}$

$= (Cb-\gamma)'(C(X'X)^{-1}\underbrace{A'^{-1}A'}_{I}C')^{-1}(Cb-\gamma)/(ms^2)$

$= \dfrac{(Cb-\gamma)'(C(X'X)^{-1}C')^{-1}(Cb-\gamma)}{ms^2}$ same $F^*$