

STA 302 Formulas

$$M_y(t) = E(e^{yt})$$

$$M_{y+a}(t) = e^{at} M_y(t)$$

$$y \sim N(\mu, \sigma^2) \text{ means } M_y(t) = e^{\mu t + \frac{1}{2} \sigma^2 t^2}$$

If $w = w_1 + w_2$ with w_1 and w_2 independent, $w \sim \chi^2(\nu_1 + \nu_2)$, $w_2 \sim \chi^2(\nu_2)$ then $w_1 \sim \chi^2(\nu_1)$

$$y_i = \beta_0 + \beta_1 x_i + \epsilon_i$$

$$b_1 = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^n (x_i - \bar{x})^2} = \frac{\sum_{i=1}^n x_i y_i - n \bar{x} \bar{y}}{\sum_{i=1}^n x_i^2 - n \bar{x}^2}$$

Columns of A linearly dependent means there is a vector $\mathbf{v} \neq \mathbf{0}$ with $A\mathbf{v} = \mathbf{0}$.

A positive semi-definite means $\mathbf{v}' A \mathbf{v} \geq 0$.

$$\Sigma = CDC'$$

$$\Sigma^{1/2} = CD^{1/2}C'$$

$$\text{cov}(\mathbf{y}) = E\{(\mathbf{y} - \boldsymbol{\mu}_y)(\mathbf{y} - \boldsymbol{\mu}_y)'\}$$

$$\text{cov}(\mathbf{y}) = E\{\mathbf{y}\mathbf{y}'\} - \boldsymbol{\mu}_y \boldsymbol{\mu}_y'$$

$$M_{\mathbf{y}}(\mathbf{t}) = E(e^{\mathbf{t}'\mathbf{y}})$$

$$M_{\mathbf{y}+\mathbf{c}}(\mathbf{t}) = e^{\mathbf{t}'\mathbf{c}} M_{\mathbf{y}}(\mathbf{t})$$

\mathbf{y}_1 and \mathbf{y}_2 are independent if and only if $M_{(\mathbf{y}_1, \mathbf{y}_2)}(\mathbf{t}_1, \mathbf{t}_2) = M_{\mathbf{y}_1}(\mathbf{t}_1)M_{\mathbf{y}_2}(\mathbf{t}_2)$

If $\mathbf{y} \sim N_p(\boldsymbol{\mu}, \Sigma)$, then $A\mathbf{y} + \mathbf{c} \sim N_q(A\boldsymbol{\mu} + \mathbf{c}, A\Sigma A')$,

$$y_i = \beta_0 + \beta_1 x_{i1} + \dots + \beta_k x_{ik} + \epsilon_i$$

$$\mathbf{y} = X\boldsymbol{\beta} + \boldsymbol{\epsilon} \text{ with } \boldsymbol{\epsilon} \sim N(\mathbf{0}, \sigma^2 I_n)$$

$$\hat{\mathbf{y}} = X\hat{\mathbf{b}} = H\mathbf{y}, \text{ where } H = X(X'X)^{-1}X'$$

\mathbf{b} and \mathbf{e} are independent under normality.

$$\sum_{i=1}^n (y_i - \bar{y})^2 = \sum_{i=1}^n (y_i - \hat{y}_i)^2 + \sum_{i=1}^n (\hat{y}_i - \bar{y})^2$$

$$t = \frac{Z}{\sqrt{W/\nu}} \sim t(\nu)$$

$$t = \frac{\boldsymbol{\ell}'\mathbf{b} - \boldsymbol{\ell}'\boldsymbol{\beta}}{s\sqrt{\boldsymbol{\ell}'(X'X)^{-1}\boldsymbol{\ell}}} \sim t(n - k - 1)$$

$$F = \frac{(\mathbf{Cb} - \boldsymbol{\gamma})'(\mathbf{C}(X'X)^{-1}\mathbf{C}')^{-1}(\mathbf{Cb} - \boldsymbol{\gamma})}{m s^2}$$

$$s^2 = \frac{SSE}{n - k - 1} = \frac{\mathbf{e}'\mathbf{e}}{n - k - 1}$$

$$t = \frac{y_0 - \mathbf{x}'_0 \mathbf{b}}{s\sqrt{1 + \mathbf{x}'_0 (X'X)^{-1} \mathbf{x}_0}} \sim t(n - k - 1)$$

$$M_{ay}(t) = M_y(at)$$

$$M_{\sum_{i=1}^n y_i}(t) = \prod_{i=1}^n M_{y_i}(t)$$

$$y \sim \chi^2(\nu) \text{ means } M_y(t) = (1 - 2t)^{-\nu/2}$$

$$b_0 = \bar{y} - b_1 \bar{x}$$

$$r = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^n (x_i - \bar{x})^2} \sqrt{\sum_{i=1}^n (y_i - \bar{y})^2}}$$

Columns of A linearly independent means that $A\mathbf{v} = \mathbf{0}$ implies $\mathbf{v} = \mathbf{0}$.

A positive definite means $\mathbf{v}' A \mathbf{v} > 0$ if $\mathbf{v} \neq \mathbf{0}$.

$$\Sigma^{-1} = CD^{-1}C'$$

$$\Sigma^{-1/2} = CD^{-1/2}C'$$

$$\text{cov}(\mathbf{y}, \mathbf{t}) = E\{(\mathbf{y} - \boldsymbol{\mu}_y)(\mathbf{t} - \boldsymbol{\mu}_t)'\}$$

$$\text{cov}(A\mathbf{y}) = A\text{cov}(\mathbf{y})A'$$

$$M_{A\mathbf{y}}(\mathbf{t}) = M_{\mathbf{y}}(A'\mathbf{t})$$

$$\mathbf{y} \sim N_p(\boldsymbol{\mu}, \Sigma) \text{ means } M_{\mathbf{y}}(\mathbf{t}) = e^{\mathbf{t}'\boldsymbol{\mu} + \frac{1}{2}\mathbf{t}'\Sigma\mathbf{t}}$$

and $w = (\mathbf{y} - \boldsymbol{\mu})'\Sigma^{-1}(\mathbf{y} - \boldsymbol{\mu}) \sim \chi^2(p)$

$\epsilon_1, \dots, \epsilon_n$ independent $N(0, \sigma^2)$

$$\mathbf{b} = (X'X)^{-1}X'\mathbf{y} \sim N_{k+1}(\boldsymbol{\beta}, \sigma^2(X'X)^{-1})$$

$$\mathbf{e} = \mathbf{y} - \hat{\mathbf{y}} = (I - H)\mathbf{y}$$

$$\frac{SSE}{\sigma^2} = \frac{\mathbf{e}'\mathbf{e}}{\sigma^2} \sim \chi^2(n - k - 1)$$

$$SST = SSE + SSR \text{ and } R^2 = \frac{SSR}{SST}$$

$$F = \frac{W_1/\nu_1}{W_2/\nu_2} \sim F(\nu_1, \nu_2)$$

$$\boldsymbol{\ell}'\mathbf{b} \pm t_{\alpha/2} s\sqrt{\boldsymbol{\ell}'(X'X)^{-1}\boldsymbol{\ell}}$$

$$F = \frac{SSR_F - SSR_R}{m s^2} = \left(\frac{n-k-1}{m}\right) \left(\frac{a}{1-a}\right)$$

$$a = \frac{R_F^2 - R_R^2}{1 - R_R^2} = \frac{mF}{n - k - 1 + mF}$$

$$\mathbf{x}'_0 \mathbf{b} \pm t_{\alpha/2} s\sqrt{1 + \mathbf{x}'_0 (X'X)^{-1} \mathbf{x}_0}$$

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<http://www.utstat.toronto.edu/~brunner/oldclass/302f16>