Name Jerry

Student Number

STA 302f 2015 Quiz 8

1. (4 points) Just give the answers to the questions below. Only re-derive the the distributions if you can't remember.

   (a) Let $Y \sim N(\mu, \sigma^2)$ and $W = \frac{X-\mu}{\sigma}$. What is the distribution of W?

   $W \sim N(0,1)$

   (b) Let $Y_1, \ldots, Y_n$ be independent $N(\mu, \sigma^2)$ random variables. What is the distribution of $W = \frac{\sqrt{n}(\overline{X}-\mu)}{\sigma}$?

   $W \sim N(0,1)$

   (c) Let $Y \sim N(0,1)$. What is the distribution of $W = Y^2$?

   $W \sim \chi^2(1)$

   (d) Let $Y_1, \ldots, Y_n$ be independent $\chi^2(1)$ random variables. What is the distribution of $W = \sum_{i=1}^{n} Y_i$?

   $W \sim \chi^2(n)$

2. (6 points) *Answer True or False* and show your work. For the general linear regression model with normal error terms,

$$Z = \frac{\mathbf{a}'\widehat{\boldsymbol{\beta}} - \mathbf{a}'\boldsymbol{\beta}}{\sqrt{\sigma^2\,\mathbf{a}'(\mathbf{X}'\mathbf{X})^{-1}\mathbf{a}}}$$

has a standard normal distribution.

$$a'\hat{\beta} \sim N(a'\beta,\ a'\operatorname{cov}(\hat{\beta})a)$$

$$\operatorname{cov}(\hat{\beta}) = \operatorname{cov}\left((X'X)^{-1}X'y\right) = (X'X)^{-1}X'\sigma^2 I\left((X'X)^{-1}X'\right)'$$

$$= \sigma^2 (X'X)^{-1}\underbrace{X'X(X'X)^{-1}}_{I} = \sigma^2 (X'X)^{-1}$$

If they just remember $\operatorname{cov}(\hat{\beta})$ no marks off

So $a'\hat{\beta} \sim N(a'\beta,\ \sigma^2 a'(X'X)^{-1}a)$, and

$$Z = \frac{a'\hat{\beta} - a'\beta}{\sqrt{\sigma^2 a'(X'X)^{-1}a}} \sim N(0,1)$$

TRUE