Name Jerry

Student Number

# STA 302 f2015 Quiz 1

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|---|---|
| $E(X) = \sum_x p_X(x)$ | $E(X) = \int_{-\infty}^{\infty} x f_X(x)\, dx$ |
| $E(g(X)) = \sum_x g(x)\, p_X(x)$ | $E(g(\mathbf{X})) = \sum_{x_1} \cdots \sum_{x_p} g(x_1, \ldots, x_p)\, p_{\mathbf{X}}(x_1, \ldots, x_p)$ |
| $E(g(X)) = \int_{-\infty}^{\infty} g(x)\, f_X(x)\, dx$ | $E(g(\mathbf{X})) = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} g(x_1, \ldots, x_p)\, f_{\mathbf{X}}(x_1, \ldots, x_p)\, dx_1 \ldots dx_p$ |
| $E(\sum_{i=1}^n a_i X_i) = \sum_{i=1}^n a_i E(X_i)$ | $Var(X) = E\left( (X - \mu_X)^2 \right)$ |
| $Cov(X,Y) = E\left( (X - \mu_X)(Y - \mu_Y) \right)$ | $Corr(X,Y) = \frac{Cov(X,Y)}{\sqrt{Var(X)Var(Y)}}$ |

1. (5 points) Let $X$ and $Y$ be random variables, with $E(X) = \mu_x$, $E(Y) = \mu_y$, $Var(X) = \sigma_x^2$, $Var(Y) = \sigma_y^2$, $Cov(X,Y) = \sigma_{xy}$ and $Corr(X,Y) = \rho_{xy}$. Let $a$, $b$, $c$ and $d$ be constants. Find $Cov(aX + b, cY + d)$. Show your work.

$E(aX+b) = a\mu_x + b$, $E(cY+d) = c\mu_y + d$, so

$Cov(aX+b, cY+d)$

$= E\{(aX+b-(a\mu_x+b))(cY+d-(c\mu_y+d))\}$

$= E\{a(X-\mu_x)c(Y-\mu_y)\}$

$= ac\,E\{(X-\mu_x)(Y-\mu_y)\}$

$= ac\,\sigma_{xy}$

2. (3 points) Let $\mathbf{A}$ and $\mathbf{B}$ be $2 \times 2$ matrices. Give a simple numerical example in which $\mathbf{AB} \neq \mathbf{BA}$. Carry out the multiplication in both orders, showing $\mathbf{AB} \neq \mathbf{BA}$.

Let $A = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$

$AB = \begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix}$, $BA = \begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix}$

$AB \neq BA$

3. (2 points) Let $\mathbf{X}$ be an $n$ by $p$ matrix with $n \neq p$. Why is it incorrect to say that $(\mathbf{X}'\mathbf{X})^{-1} = \mathbf{X}^{-1}\mathbf{X}'^{-1}$?

X is not a square matrix, so the inverse is not defined.