Name Jerry

Student Number

STA 302 f2014 Quiz 3A

1. (3 points) Let the $p \times 1$ random vector $\mathbf{X}$ have mean $\boldsymbol{\mu}$ and variance-covariance matrix $\boldsymbol{\Sigma}$, and let $\mathbf{c}$ be a $p \times 1$ vector of constants. Either (a) $cov(\mathbf{X}+\mathbf{c}) = \boldsymbol{\Sigma}+\mathbf{c}$, or (b) $cov(\mathbf{X}+\mathbf{c}) = \boldsymbol{\Sigma}$. Choose one and prove it.

$E(X+c) = \mu + c$, so

$$cov(X+c) = E\{(X+c-(\mu+c))(X+c-(\mu+c))'\}$$

$$= E\{(X+\cancel{c}-\mu-\cancel{c})(X+\cancel{c}-\mu-\cancel{c})'\}$$

$$= E\{(X-\mu)(X-\mu)'\} = \Sigma$$

2. (3 points) Suppose the matrix **A** has an inverse. Prove that the columns of **A** are linearly independent. This is quick.

$$A v = 0 \Rightarrow A^{-1} A v = A^{-1} 0 = 0$$

$$\Rightarrow v = 0$$

3. (4 points) For homework (Question 1, Problem 2.14g in the text), you were asked to calculate $\mathbf{xx}'$. Copy the answer into the space below. Attach the R printout, and **Circle the answer on the printout.** Make sure your name is on the printout.

$$xx' = \begin{pmatrix} 9 & -3 & 6 \\ -3 & 1 & -2 \\ 6 & -2 & 4 \end{pmatrix}$$

```
> # 2.14g
> x = c(3,-1,2)
> x %*% t(x)
     [,1] [,2] [,3]
[1,]    9   -3    6
[2,]   -3    1   -2
[3,]    6   -2    4
```

Name Jerry

Student Number

STA 302 f2014 Quiz 3B

1. (3 points) Let the $p \times 1$ random vector $\mathbf{X}$ have expected value μ variance-covariance matrix $\mathbf{\Sigma}$ and let $\mathbf{A}$ be an $m \times p$ matrix of constants. Either (a) $cov(\mathbf{AX}) = \mathbf{A\Sigma A'}$ or (b) $cov(\mathbf{AX}) = \mathbf{A'\Sigma A}$. Choose one and prove it. You may use the formula sheet for a definition, but not directly for the result.

$E(AX) = AE(X) = A\mu$, so

$$
\begin{aligned}
cov(AX) &= E\{(AX - A\mu)(AX - A\mu)'\} \\
&= E\{A(X-\mu)(A(X-\mu))'\} \\
&= E\{A(X-\mu)(X-\mu)'A'\} \\
&= A\,E\{(X-\mu)(X-\mu)'\}A' \\
&= A\Sigma A'
\end{aligned}
$$

2. (3 points) The (square) matrix $\mathbf{\Sigma}$ is said to be *positive definite* if $\mathbf{v}'\mathbf{\Sigma}\mathbf{v} > 0$ for all vectors $\mathbf{v} \neq \mathbf{0}$. Show that the eigenvalues of a positive definite matrix are all strictly positive. Hint: start with the definition of an eigenvalue and the corresponding eigenvalue: $\mathbf{\Sigma}\mathbf{v} = \lambda\mathbf{v}$.

$$\Sigma v = \lambda v \implies v'\Sigma v = v'\lambda v = \lambda v'v > 0$$

because Σ is positive definite.

Since $v \neq 0$, $v'v > 0$, and

$$\frac{\lambda v'v}{v'v} > \frac{0}{v'v} \implies \lambda > 0$$

Maybe only 1/2 point for this part

3. (4 points) For homework (Question 1, Problem 2.14m in the text), you were asked to calculate $\mathbf{C}'\mathbf{C}$. Copy the answer into the space below. Attach the R printout, and **Circle the answer on the printout.** Make sure your name is on the printout.

$$C'C = \begin{pmatrix} 14 & -7 \\ -7 & 26 \end{pmatrix}$$

```
> # 2.14m
> C = rbind(c(2,-3),
+           c(-1,4),
+           c(3,1) )
> t(C) %*% C
     [,1] [,2]
[1,]   14   -7
[2,]   -7   26
```