Name Jerry

Student Number

STA 302 f2014 Quiz 1A

$E(g(X)) = \int_{-\infty}^{\infty} g(x)\, f_X(x)\, dx \quad E(g(\mathbf{X})) = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} g(x_1, \ldots, x_p)\, f_{\mathbf{X}}(x_1, \ldots, x_p)\, dx_1 \ldots dx_p$

$Var(Y) = E[(Y - \mu_Y)^2] \quad Cov(X, Y) = E[(X - \mu_X)(Y - \mu_Y)]$

1. (5 points) Let $Y_1, \ldots, Y_n$ be independent random variables with $E(Y_i) = \mu$ and $Var(Y_i) = \sigma^2$ for $i = 1, \ldots, n$. Let $a_1, \ldots, a_n$ be constants and define the linear combination L by $L = \sum_{i=1}^n a_i Y_i$. Recall that a statistic T is an *unbiased estimator* of a parameter θ if $E(T) = \theta$ for all θ. Suppose that $\sum_{i=1}^n a_i = 2$. **Is L an unbiased estimator of μ? Answer Yes or No and show your work.** Use familiar properties of expected value, not integrals.

$$E(L) = E\left(\sum_{i=1}^n a_i Y_i\right) = \sum_{i=1}^n a_i E(Y_i)$$

$$= \sum_{i=1}^n a_i \mu = \mu \sum_{i=1}^n a_i = 2\mu \neq \mu$$

in general

So No

2. (3 points) Circle the correct answer in each multiple choice question below. You must get at least 4 out of 5 right to get any marks on this part. Quantities in boldface are matrices of constants, while letters like a are real numbers.

   (a) Which statement is true?

   i. $\mathbf{A}(\mathbf{B}+\mathbf{C}) = \mathbf{AB}+\mathbf{AC}$

   ii. $\mathbf{A}(\mathbf{B}+\mathbf{C}) = \mathbf{BA}+\mathbf{CA}$

   iii. Both ~~a~~ i and ~~b~~ ii

   iv. Neither ~~a~~ i nor ~~b~~ ii

   (b) Which statement is true?

   i. $a(\mathbf{B}+\mathbf{C}) = a\mathbf{B}+a\mathbf{C}$

   ii. $a(\mathbf{B}+\mathbf{C}) = \mathbf{B}a+\mathbf{C}a$

   iii. Both ~~a~~ i and ~~b~~ ii

   iv. Neither ~~a~~ i nor ~~b~~ ii

   (c) Which statement is true?

   i. $(\mathbf{B}+\mathbf{C})\mathbf{A} = \mathbf{AB}+\mathbf{AC}$

   ii. $(\mathbf{B}+\mathbf{C})\mathbf{A} = \mathbf{BA}+\mathbf{CA}$

   iii. Both ~~a~~ i and ~~b~~ ii

   iv. Neither a nor b

   (d) Which statement is true?

   i. $(\mathbf{AB})' = \mathbf{A}'\mathbf{B}'$

   ii. $(\mathbf{AB})' = \mathbf{B}'\mathbf{A}'$

   iii. Both ~~a~~ i and ~~b~~ ii

   iv. Neither a nor b

   (e) Which statement is true?

   i. $\mathbf{A}'' = \mathbf{A}$

   ii. $\mathbf{A}''' = \mathbf{A}'$

   iii. Both ~~a~~ i and ~~b~~ ii

   iv. Neither ~~a~~ i nor ~~b~~ ii

3. (2 points) Let $\mathbf{X}$ be an n by p matrix with $n \neq p$. Why is it incorrect to say that $(\mathbf{X}'\mathbf{X})^{-1} = \mathbf{X}^{-1}\mathbf{X}'^{-1}$?

X is not a square matrix, so the inverse is not defined.

Name Jerry

Student Number

STA 302 f2014 Quiz 1B

$E(g(X)) = \int_{-\infty}^{\infty} g(x)\, f_X(x)\, dx \quad E(g(\mathbf{X})) = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} g(x_1, \ldots, x_p)\, f_{\mathbf{X}}(x_1, \ldots, x_p)\, dx_1 \ldots dx_p$

$Var(Y) = E[(Y - \mu_Y)^2] \quad Cov(X,Y) = E[(X - \mu_X)(Y - \mu_Y)]$

1. (5 points) Let X and Y be random variables with $E(X) = E(Y) = 0$. **Circle** one of the statements below and prove it is true. Use properties of expected value, not integrals. No marks if you assume independence.

   (a) $Var(X+Y) = Var(X)Var(Y)$

   (b) $Var(X+Y) = 0$

   (c) $Var(X+Y) = Var(X) + Var(Y) + Cov(X,Y)$

   (d) $Var(X+Y) = Var(X) + Var(Y) + 2Cov(X,Y)$

   (e) $Var(X+Y) = Var(X) + Var(Y)$

$Var(X+Y) = E\{(X+Y-0)^2\} = E(X^2 + 2XY + Y^2)$

$= E(X^2) + E(Y^2) + 2E(XY)$

$= Var(X) + Var(Y) + 2\,cov(X,Y)$

2. (3 points) Label each statement below True or False. Write "T" or "F" beside each statement. Assume the $\alpha = 0.05$ significance level. You must get at least 4 out of 5 right to get any marks on this part.

(a) F The p-value is the probability that the null hypothesis is true.

(b) F The p-value is the probability that the null hypothesis is false.

(c) F In a study comparing a new drug to the current standard treatment, the null hypothesis is rejected. This means the new drug is ineffective.

(d) F The p-value is the probability of failing to replicate significant results in a second independent random sample of the same size.

(e) F If $p > .05$ we reject the null hypothesis at the .05 level.

3. (2 points) Let $\mathbf{X}$ be an n by p matrix with $n \neq p$. Why is it incorrect to say that $(\mathbf{X}'\mathbf{X})^{-1} = \mathbf{X}^{-1}\mathbf{X}'^{-1}$? You have more room than you need.

X is not a square matrix, so the inverse is not defined.