Family (Last) Name _____

Given (First) Name Jerry

Student Number _____

STA 302s13 Quiz 4A

1. (6 points) The simple linear regression model is $Y_i = \beta_0 + \beta_1 x_i + \epsilon_i$ for $i = 1, \ldots, n$, where $\epsilon_1, \ldots, \epsilon_n$ are a random sample from a distribution with expected value zero and variance σ^2. The numbers $x_1, \ldots, x_n$ are known, observed constants, while the parameters β_0, β_1 and σ^2 are unknown constants (parameters).

   (a) Differentiate $Q(\beta_0, \beta_1) = \sum_{i=1}^{n}(Y_i - \beta_0 - \beta_1 x_i)^2$ with respect to β_0 and set the derivative to zero, obtaining the first normal equation. Circle your answer

$$\frac{\partial Q}{\partial \beta_0} = \sum_{i=1}^{n} \frac{\partial}{\partial \beta_0}(Y_i - \beta_0 - \beta_1 x_i)^2 = \sum_{i=1}^{n} 2(Y_i - \beta_0 - \beta_1 x_i)(-1)$$

$$\stackrel{\text{set}}{=} 0 \Rightarrow \sum_{i=1}^{n} Y_i - n\beta_0 - \beta_1 \sum_{i=1}^{n} x_i = 0$$ This is OKAY

$$\Rightarrow \sum_{i=1}^{n} Y_i = n\beta_0 - \beta_1 \sum_{i=1}^{n} x_i$$

   (b) Noting that the quantities $\widehat{\beta}_0$ and $\widehat{\beta}_1$ must satisfy the first normal equation and defining "predicted" Y_i as $\widehat{Y}_i = \widehat{\beta}_0 + \widehat{\beta}_1 x_i$, show that $\sum_{i=1}^{n} \widehat{Y}_i = \sum_{i=1}^{n} Y_i$. There is more room on the next page if you need it.

From the first normal equation,

$$\sum_{i=1}^{n} Y_i = n\hat{\beta}_0 - \hat{\beta}_1 \sum_{i=1}^{n} x_i = \sum_{i=1}^{n}(\hat{\beta}_0 - \hat{\beta}_1 x_i) = \sum_{i=1}^{n} \hat{Y}_i$$

2. (4 points) Please attach your R printout from Question 2(g)vii to the quiz. *Circle* $\widehat{\beta}_0$ *and* $\widehat{\beta}_1$ *on the printout.* **If the correct numbers are not circled, you get a zero**. Please make sure your name appears on the printout.

```
R version 2.15.1 (2012-06-22) -- "Roasted Marshmallows"
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Platform: i386-apple-darwin9.8.0/i386 (32-bit)

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[R.app GUI 1.52 (6188) i386-apple-darwin9.8.0]

[Workspace restored from /Users/brunner/.RData]
[History restored from /Users/brunner/.Rapp.history]

> x = c(1, 8, 3, 6, 4, 7)
> y = c(14, 2, 14, 10, 9, 9)
> lsfit(x,y)
$coefficients
Intercept         X
16.511962 -1.416268

$residuals
[1] -1.095694 -3.181818  1.736842  1.985646 -1.846890  2.401914

$intercept
[1] TRUE

$qr
$qt
[1] -23.678401   8.358781   1.851326   2.944197  -1.451050   3.641821

$qr
     Intercept            X
[1,] -2.4494897 -11.83920042
[2,]  0.4082483  -5.90197707
[3,]  0.4082483  -0.12234169
[4,]  0.4082483   0.38596255
[5,]  0.4082483   0.04709306
[6,]  0.4082483   0.55539730

$qraux
[1] 1.408248 1.724832

$rank
[1] 2

$pivot
[1] 1 2

$tol
[1] 1e-07

attr(,"class")
[1] "qr"

>
```

Family (Last) Name _____

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STA 302s13 Quiz 4B

1. (2 points) The $p \times p$ matrix $\mathbf{A}$ is said to be *non-negative definite* if $\mathbf{v}'\mathbf{A}\mathbf{v} \geq 0$ for all constant vectors $\mathbf{v} \in \mathbb{R}^p$. Show that $\mathbf{X}'\mathbf{X}$ is non-negative definite, where $\mathbf{X}$ is the $n \times (k+1)$ constant matrix from a linear regression model.

$$v'X'Xv = (Xv)'Xv = z'z = \sum_{i=1}^{n} z_i^2 \geq 0$$

2. (4 points) Recall the definition of linear dependence. The columns of $\mathbf{A}$ are said to be *linearly dependent* if there exists a column vector $\mathbf{v} \neq \mathbf{0}$ with $\mathbf{A}\mathbf{v} = \mathbf{0}$. Show that if the columns of $\mathbf{X}$ are linearly dependent, then $(\mathbf{X}'\mathbf{X})^{-1}$ cannot exist. There is more room on the next page if you need it.

By linear dependence, there is $v \neq 0$ with

$Xv = 0 \Rightarrow X'Xv = 0$.

If $(X'X)^{-1}$ existed, one could continue

$\Rightarrow \underbrace{(X'X)^{-1}X'X}_{I} v = (X'X)^{-1}0 = 0$

$\Rightarrow v = 0$. But $v \neq 0$. This contradiction shows $(X'X)^{-1}$ cannot exist. ▨

3. (4 points) Please attach your R printout from Question 2(g)vii to the quiz. *Circle and label* $\widehat{\beta}_0$ *and* $\widehat{\beta}_1$ *on the printout.* **If the correct numbers are not circled *and labelled*, you get a zero**. Please make sure your name appears on the printout.

```
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[Workspace restored from /Users/brunner/.RData]
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