Name Jerry

Student Number

STA 302s13 Quiz 1[redacted]A

$E(g(X)) = \int_{-\infty}^{\infty} g(x)\, f_X(x)\, dx$, or $E(g(X)) = \sum_x g(x)\, p_X(x)$

$Var(X) = E[(X - \mu_X)^2]$ $\quad$ $Cov(X, Y) = E[(X - \mu_X)(Y - \mu_Y)]$

with $E(X) = \mu_x$ and $E(Y) = \mu_y$

1. (5 points) In this question, X and Y are random variables, while a and b are constants. Circle the letter corresponding to the correct statement, and prove it. You may well-known properties of expected value without proof.

   (a) $Cov(X + a, Y + b) = Var(X) + Var(Y) + 2abCov(X, Y)$

   (b) $Cov(X + a, Y + b) = Cov(X, Y)$

   (c) $Cov(X + a, Y + b) = 0$

$E(X+a) = \mu_x + a$, $E(Y+b) = \mu_y + b$

$$Cov(X+a, Y+b) = E\{(X+a-(\mu_x+a))(Y+b-(\mu_y+b))\}$$

$$= E\{(X+\cancel{a}-\mu_x-\cancel{a})(Y+\cancel{b}-\mu_y-\cancel{b})\}$$

$$= E\{(X-\mu_x)(Y-\mu_y)\}$$

$$= Cov(X, Y)$$

2. (5 points) Let $Y_1, \ldots, Y_n$ be a random sample from a normal distribution with mean μ and variance σ^2, so that $T = \frac{\sqrt{n}(\overline{Y}-\mu)}{S} \sim t(n-1)$. This is something you may use without proof. Derive a $(1-\alpha)100\%$ confidence interval for μ. "Derive" means show all the high school algebra. Use the symbol $t_{\alpha/2}$ for the number satisfying $Pr(T > t_{\alpha/2}) = \alpha/2$.

$$1-\alpha = P\left(-t_{\alpha/2} < T < t_{\alpha/2}\right)$$

$$= P\left(-t_{\alpha/2} < \frac{\sqrt{n}(\overline{Y}-\mu)}{S} < t_{\alpha/2}\right)$$

$$= P\left(-t_{\alpha/2}\frac{S}{\sqrt{n}} < \overline{Y}-\mu < t_{\alpha/2}\frac{S}{\sqrt{n}}\right)$$

$$= P\left(-\overline{Y}-t_{\alpha/2}\frac{S}{\sqrt{n}} < -\mu < -\overline{Y}+t_{\alpha/2}\frac{S}{\sqrt{n}}\right)$$

$$= P\left(\overline{Y}+t_{\alpha/2}\frac{S}{\sqrt{n}} > \mu > \overline{Y}-t_{\alpha/2}\frac{S}{\sqrt{n}}\right)$$

$$= P\left(\overline{Y}-t_{\alpha/2}\frac{S}{\sqrt{n}} < \mu < \overline{Y}+t_{\alpha/2}\frac{S}{\sqrt{n}}\right)$$ This step optional

So the confidence interval is

$$\left(\overline{Y}-t_{\alpha/2}\frac{S}{\sqrt{n}},\ \overline{Y}+t_{\alpha/2}\frac{S}{\sqrt{n}}\right),$$ or

$$\overline{Y} \pm t_{\alpha/2}\frac{S}{\sqrt{n}}$$

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STA 302s13 Quiz 1B

$E(g(X)) = \int_{-\infty}^{\infty} g(x)\, f_X(x)\, dx,$ or $E(g(X)) = \sum_x g(x)\, p_X(x)$

$Var(X) = E[(X - \mu_X)^2]$ $Cov(X,Y) = E[(X - \mu_X)(Y - \mu_Y)]$

1. (5 points) Let X and Y be random variables with $E(X) = \mu_x$ and $E(Y) = \mu_y$. Calculate $Var(X+Y)$ in terms of variances and covariances. **Show your work**. You may use well-known properties of expected value without proof.

$E(X+Y) = \mu_x + \mu_y$, so

$$Var(X+Y) = E\{(X+Y-(\mu_x+\mu_y))^2\}$$

$$= E\{(X-\mu_x + Y-\mu_y)^2\}$$

$$= E\{(X-\mu_x)^2 + 2(X-\mu_x)(Y-\mu_y) + (Y-\mu_y)^2\}$$

$$= E\{(X-\mu_x)^2\} + 2E\{(X-\mu_x)(Y-\mu_y)\} + E\{(Y-\mu_y)^2\}$$

$$= Var(X) + 2\,Cov(X,Y) + Var(Y)$$

2. (5 points) Recall that an inverse of the square matrix $\mathbf{A}$ (denoted $\mathbf{A}^{-1}$) is defined by two properties: $\mathbf{A}^{-1}\mathbf{A} = \mathbf{I}$ and $\mathbf{A}\mathbf{A}^{-1} = \mathbf{I}$. Prove that inverses are unique, as follows. Let $\mathbf{B}$ and $\mathbf{C}$ both be inverses of $\mathbf{A}$. Show that $\mathbf{B} = \mathbf{C}$. This is fast if you do it the right way.

$$AB = I$$

$$\Rightarrow \underbrace{CA}_{I}B = CI = C$$

$$\Rightarrow B = C \quad \square$$