

Random Independent Variables

STA302 Fall 2013

[See last slide for copyright information](#)

Don't you think it's strange?

- In the general linear regression model, the X matrix is supposed to be full of fixed constants.
- But in any non-experimental study, if you selected another sample, you'd get different X values, because of random sampling.
- So X should be random variables, not fixed.
- View the usual model as *conditional* on $X=x$.

Recall Double Expectation

$$E\{Y\} = E\{E\{Y|X\}\}$$

$E\{Y\}$ is a constant. $E\{Y|X\}$ is a random variable, a function of X .

$$E\{E\{Y|X\}\} = \int E\{Y|X = x\} f(x) dx$$

Beta-hat is (conditionally) unbiased

$$E\{\hat{\beta} | \mathbf{X} = \mathbf{x}\} = \beta$$

Unbiased unconditionally, too

$$E\{\hat{\beta}\} = E\{E\{\hat{\beta} | \mathbf{X}\}\} = E\{\beta\} = \beta$$

Perhaps Clearer

$$\begin{aligned} E\{\hat{\boldsymbol{\beta}}\} &= E\{E\{\hat{\boldsymbol{\beta}}|\mathbf{X}\}\} \\ &= \int \cdots \int E\{\hat{\boldsymbol{\beta}}|\mathbf{X} = \mathbf{x}\} f(\mathbf{x}) d\mathbf{x} \\ &= \int \cdots \int \boldsymbol{\beta} f(\mathbf{x}) d\mathbf{x} \\ &= \boldsymbol{\beta} \int \cdots \int f(\mathbf{x}) d\mathbf{x} \\ &= \boldsymbol{\beta} \cdot 1 = \boldsymbol{\beta}. \end{aligned}$$

Conditional size α test, Critical region A

$$\Pr\{F \in A | \mathbf{X} = \mathbf{x}\} = \alpha$$

$$\begin{aligned}\Pr\{F \in A\} &= \int \cdots \int \Pr\{F \in A | \mathbf{X} = \mathbf{x}\} f(\mathbf{x}) d\mathbf{x} \\ &= \int \cdots \int \alpha f(\mathbf{x}) d\mathbf{x} \\ &= \alpha \int \cdots \int f(\mathbf{x}) d\mathbf{x} \\ &= \alpha\end{aligned}$$

The moral of the story

- Don't worry.
- Even though X variables are often random, we can apply the usual fixed- x model without fear.
- Estimators are still unbiased.
- Tests have the right significance level.
- And it's all *distribution-free* with respect to X .

Copyright Information

This slide show was prepared by Jerry Brunner, Department of Statistics, University of Toronto. It is licensed under a Creative Commons Attribution - ShareAlike 3.0 Unported License. Use any part of it as you like and share the result freely. These Powerpoint slides will be available from the course website: <http://www.utstat.toronto.edu/brunner/oldclass/302f13>