STA 261s2006 Assignment 9

Do this assignment in preparation for the quiz on Wednesday, March 15th. The questions are practice for the quiz, and are not to be handed in.

1. Let $X_1, \ldots, X_n$ be a random sample from a normal distribution with expected value μ and variance σ^2.

   (a) Given that $\overline{X}$ and S^2 are independent, prove that $\frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1)$.

   (b) Using Question 1a, derive an exact $(1-\alpha)100\%$ confidence interval for σ^2. Then do Exercise 11.61; assume normality. You need not use a computer program; a calculator will be fine.

   (c) Using Question 1a and the independence of $\overline{X}$ and S^2, derive an exact $(1-\alpha)100\%$ confidence interval for μ. Then do Exercise 11.31; assume normality.

2. Let $X_1, \ldots, X_{n_1}$ be a random sample from a $N(\mu_1, \sigma^2)$ distribution, and let $Y_1, \ldots, Y_{n_2}$ be a random sample from a $N(\mu_2, \sigma^2)$ distribution. These are *independent* random samples, meaning that the X and Y values are independent. Notice that while the population means μ_1 and μ_2 may be different, the population variances are the same.

   (a) Show that the random variable Z defined on page 360 has a standard normal distribution.

   (b) What is the distribution of $\frac{(n_1-1)S_1^2+(n_2-1)S_2^2}{\sigma^2}$? Prove your answer, assuming your answer to Question 1a.

   (c) Prove that the random variable T defined on page 360 has a t distribution.

   (d) Derive an exact $(1-\alpha)100\%$ confidence interval for $\mu_1 - \mu_2$.

   (e) Do Exercise 11.35

3. Let $X_1, \ldots, X_{n_1}$ be a random sample from a distribution (not necessarily normal) with expected value μ_1 and variance σ_1^2, and let $Y_1, \ldots, Y_{n_2}$ be a random sample from a distribution (not necessarily normal) with expected value μ_2 and variance σ_2^2. The random samples are independent of each other. The Central Limit Theorem tells us that for large n_1, the distribution of $\overline{X}_{n_1}$ is approximately $N(\mu_1, \frac{\sigma_1^2}{n_1})$. Similarly, the distribution of $\overline{Y}_{n_2}$ is approximately $N(\mu_2, \frac{\sigma_2^2}{n_2})$. So, what should the approximate distribution of $\overline{X}_{n_1} - \overline{Y}_{n_2}$ be?

4. Using your answer to Question 3, give an approximate large-sample confidence interval for $\mu_1 - \mu_2$. Compare Theorem 11.4. Now, to make it useable in practice, substitute consistent estimators for σ_1^2 and σ_2^2. If you don't know the distributions (the typical case, unless they are Bernoulli), you can always use S_1^2 and S_2^2. Now do Exercise 11.34 (Answ: CI for $\mu_1 - \mu_2$ is -16.3 to +1.5).

5. Read Sections 11.1 to 11.3 and do Exercises 11.6 and 11.9. For Exercise 11.9, you may use the fact that the ordinary sample variance is unbiased; don't prove it.

6. Read Sections 11.4, 11.5 and 11.6; skip 11.7. Do exercises 11.29 and 11.53.