STA 261s2006 Assignment 6

Do this assignment in preparation for Quiz 5, which will be in tutorial on Wednesday Feb. 15th. The questions are practice for the quiz, and are not to be handed in.

1. Prove $\sum_{i=1}^{n}(X_i - \overline{X})^2 = \sum_{i=1}^{n} X_i^2 - n\overline{X}^2$. This will be on the formula sheet for Test 2, but you still may need to prove it.

2. Read Section 10.5. You will not be asked to apply the definition of sufficiency – just the factorization theorem. Do Exercises 10.42, 10.43, 10.45, 10.48 and 10.49.

3. Let $X_1, \ldots, X_n$ be a random sample from a Bernoulli distribution with parameter θ.

   (a) Give a one-dimensional sufficient statistic for θ. Show your work and circle your answer.

   (b) Calculate your sufficient statistic for the following set of data: 1 0 1 0 0. Your answer is a single number; circle it. My answer is 2, but yours may be different and still correct, if you arrived at another sufficient statistic.

4. Let $X_1, \ldots, X_n$ be a random sample from a Poisson distribution with parameter λ.

   (a) Give a one-dimensional sufficient statistic for λ. In addition to being sufficient, your answer must also be an unbiased estimator. Show your work and circle your answer. You do not need to prove that your estimator is unbiased.

   (b) Calculate your sufficient statistic for the following set of data: 14 10 8 8. Your answer is a single number; circle it. My answer is 10.

5. Let $X_1, \ldots, X_n$ be a random sample from a Gamma distribution with parameters $\alpha = \theta$ and $\beta = 2$.

   (a) Give a one-dimensional sufficient statistic for θ.

   (b) Calculate your sufficient statistic for the following set of data: 0.706 2.154 2.367 4.039 2.155 1.678. Your answer is a single number; circle it. My answer is 52.57288, but yours may be different and still correct, if you arrived at another sufficient statistic.

6. Let $X_1, \ldots, X_n$ be a random sample from a uniform distribution with parameters α and β.

   (a) Give a two-dimensional sufficient statistic for (α, β). Show your work and circle your answer.

   (b) Calculate your sufficient statistic for the following set of data: 5.103 6.400 5.415 4.198 4.817 5.907. Your answer is a pair of numbers; circle them. My answer is (4.198, 6.4), but yours may be different and still correct, if you arrived at another sufficient statistic.

7. Let $X_1, \ldots, X_n$ be a random sample from a normal distribution with parameters μ and σ^2.

   (a) Give a two-dimensional sufficient statistic for (μ, σ^2). In addition to being sufficient, your statistics must also be unbiased estimators. Show your work and circle your answer. You do not need to prove that your estimators are unbiased.

   (b) Calculate your sufficient statistic for the following set of data: 100.3 100.6 96.5 99.3 104.1. Your answer is a pair of numbers; circle them. My answer is (100.16, 7.468).

8. Let $X_1, \ldots, X_n$ be a random sample from a distribution with density

$$f(x; \theta, \delta) = \frac{1}{\theta} e^{-\frac{x-\delta}{\theta}} I(x \geq \delta),$$

where $\theta > 0$ and δ is any real number.

   (a) Give a two-dimensional sufficient statistic for (θ, δ). Show your work and circle your answer.

   (b) Calculate your sufficient statistic for the following set of data: 11.03 10.34 11.26 10.02 10.42 10.58. Your answer is a pair of numbers; circle them. My answer is (63.65, 10.02), but yours may be different and still correct, if you arrived at another sufficient statistic.