STA 261s2006 Assignment 2

Do this assignment in preparation for the quiz in tutorial on Wednesday Jan 18. The questions are practice for the quiz, and are not to be handed in.

1. Here are some problems on moment-generating functions.

   (a) Let $M_X(t)$ be the moment-generating function of the random variable X, and let a be a constant. Show $M_{aX}(t) = M_X(at)$.

   (b) Let $X_1, \ldots, X_n$ be independent random variables with respective moment-generatng functions $M_{X_1}(t), \ldots, M_{X_n}(t)$. Let $Y = \sum_{i=1}^n X_i$. Show $M_Y(t) = \prod_{i=1}^n M_{X_i}(t)$. Be clear about where you use independence.

   (c) Let $X_1, \ldots, X_n$ be independent Exponential(θ) random variables. Find the distribution of $\overline{X}_n$. Show your work.

   (d) Let $Z \sim N(0,1)$, and let $X = Z^2$. Find the distribution of X using moment-generating functions. Show your work.

   (e) Let $X_1, \ldots, X_n$ be independent chi-square random variables with respective degree of freedom parameters $\nu_1, \ldots, \nu_n$. Find the distribution of $Y = \sum_{i=1}^n X_i$. Show your work.

   (f) Let $X_1, \ldots, X_n$ be independent Poisson random variables with respective parameters $\lambda_1, \ldots, \lambda_n$. Find the distribution of $Y = \sum_{i=1}^n X_i$. Show your work.

   (g) Let $X_1, \ldots, X_n$ be independent and identically distributed Binomial random variables with parameters m and θ. Find the distribution of $Y = \sum_{i=1}^n X_i$. Show your work.

   (h) Let $X_1, \ldots, X_n$ be independent and identically distributed Normal random variables with parameters μ and σ^2. Find the distribution of $\overline{X}_n$. Show your work.

2. Let $X_1, \ldots, X_n$ be independent random variables, all with the same expected value μ and the same variance σ^2.

   (a) Find $E[\overline{X}_n]$. Show your work. This is quick.

   (b) Find $Var[\overline{X}_n]$. Show your work. This is quick.

   (c) Find $E[S^2] = E\left[\frac{\sum_{i=1}^n (X_i - \overline{X}_n)^2}{n-1}\right]$. Show your work.

3. Let g be a non-negative function. That is $g(x) \geq 0$ for all x. For any constant a, show that $E(g(X)) \geq aPr\{g(X) \geq a\}$

   (a) For X discrete.

   (b) For X continuous.

   This is *Markov's inequality* (see lecture notes).

4. Read Section 4.4, Pages 141-143. Do exercises 4.29, 4.30, 4.32

5. Let $X_1, \ldots, X_n$ be independent random variables with $E(X_i) = \mu$ and $Var(X_i) = \sigma^2$. Combine your results from the first two parts of problem 2 with Chebyshev's inequality to show that for any constant $c > 0$,

$$\lim_{n \to \infty} Pr\{|\overline{X}_n - \mu| \geq c\} = 0$$

   This is the *Law of Large Numbers.*