STA 261s2006 Assignment 12

Do this assignment in preparation for the quiz on Wednesday, April 5th. The questions are practice for the quiz, and are not to be handed in.

1. Let $X_1, \ldots, X_n$ be a random sample from a Normal(μ, σ^2) distribution where σ^2 is *known*.

   (a) Derive an exact size α likelihood ratio test of $H_0 : \mu = \mu_0$ versus the alternative that $\mu \neq \mu_0$. Show your work.

   (b) Consider $H_0 : \mu \geq \mu_0$ versus $H_1 : \mu < \mu_0$.

      i. What is the unrestricted MLE $\widehat{\widehat{\mu}}$? You should be able to derive it if asked.

      ii. What is the *restricted* MLE $\widehat{\mu}$? Show all your work.

   (c) Derive an exact size α likelihood ratio test of H_0 versus H_1. Show your work.

2. Do Exercise 12.20. By X, they mean $\sum_{i=1}^n X_i$, where the X_is are Bernoulli with parameter θ. By "symmetry," they mean symmetry about the point $\frac{n}{2}$; show that $f(\frac{n}{2} - a) = f(\frac{n}{2} + a)$.

   I would like to add a part (d). Using the Central Limit Theorem (or the normal approximation to the binomial if you wish, which is what our authors would do), write the critical region explicitly so that it has approximate size α when the sample size is large. Don't bother with any continuity correction.

3. Do Exercise 12.21, but keep going. Add part (c): Now take logs, and study the function $g(x) = \ln x - \frac{x}{\theta_0}$. Using this, show that the critical region can be expressed as

$$C = \left\{ x_1, \ldots, x_n : \frac{2}{\theta_0} \sum_{i=1}^n x_i \leq a \text{ or } \frac{2}{\theta_0} \sum_{i=1}^n x_i \geq b \right\}.$$

   You don't have to specify a and b; it's a hard problem. In practice, people would likely use the $\alpha/2$ and $1 - \alpha/2$ critical values of a chisquare distribution with $2n$ degrees of freedom. This, like the usual two-sided test for the variance based on a normal sample, would not exactly be a likelihood ratio test. You could say it's "suggested by" the likelihood ratio test, or something.

   Using the test with equal tail areas, use the data of Exercise 12.44 to test $H_0 : \theta = 15$. Give the numerical value of the chisquare test statistic, the critical values, and state whether or not you reject the null hypothesis. Oops; the critical values you need are not in the table: $\chi^2_{0.975,40} = 24.433$, and $\chi^2_{0.025,40} = 59.342$. My chisquare value is $\frac{2}{\theta_0}\sum_{i=1}^n X_i = 70.53$.

4. Do Exercise 12.22. We did it in class a somewhat different way, also acceptable.

5. Do Exercise 12.24. Notice that they are asking only for the *statistic*, not the critical region. The critical region involves slippery a and b values as in Question 3. And in their hint, they mean Example 10.18, not 10.17.

6. Do Exercises 12.25 and 12.26.

7. Let $X_1, \ldots, X_{n_1}$ be a random sample from a $N(\mu_1, \sigma^2)$ distribution, and let $Y_1, \ldots, Y_{n_2}$ be a random sample from a $N(\mu_2, \sigma^2)$ distribution. These are *independent* random samples, meaning that the X and Y values are independent. Consider $H_0 : \mu_1 = \mu_2$ versus $H_1 : \mu_1 \neq \mu_2$. Derive formulas for the quantities $\widehat{\widehat{\mu}}_1$, $\widehat{\widehat{\mu}}_2$, $\widehat{\widehat{\sigma}}^2$, $\widehat{\mu}_1$, $\widehat{\mu}_2$ and $\widehat{\sigma}^2$. Show your work.

8. Again, we have two independent random samples from normal distributions with the same variance, but in this easier case σ^2 is *known*. And again, we want to test $H_0 : \mu_1 = \mu_2$ versus $H_1 : \mu_1 \neq \mu_2$ with an exact likelihood ratio test. Write the critical region in terms of a statistic that has a standard normal distribution under H_0.