Questions from Old tests and exams

4. (3 points) Let $X_1, \ldots, X_n$ be a random sample from a density that is uniform on $(\theta, \theta+10)$, where θ is an unknown real number. Let $\widehat{\Theta} = \overline{X} - 10$. Is $\widehat{\Theta}$ an unbiased estimator for θ? Clearly state whether it is biased or unbiased, and show your work.

5. (10 points) Let $X_1, \ldots, X_n$ be a random sample from a shifted exponential distribution. That is, $f_X(x;\theta) = e^{-(x_i-\theta)}\, I_{\{x_i \geq \theta\}}$, where the parameter $\theta > 0$. Find the maximum likelihood estimate of θ. Show your work.

6. (8 points) Let $X_1, \ldots, X_n$ be a random sample from a Normal $(0,\sigma^2)$ distribution. Find the MLE of σ^2. To save time, don't bother to check the second derivative.

7. Let $X_1, \ldots, X_n$ be a random sample from a Normal $(0,\sigma^2)$ distribution.

a) (10 points) Let $Y_i = \frac{X_i^2}{\sigma^2}$. Find the density of Y_i. You may use the symmetry of the normal distribution if necessary, but don't use any theorems about the normal distribution. Derive the result or get no marks. **Circle your answer.**

b) (2 points) Let $W = \sum_{i=1}^{n} Y_i = \sum_{i=1}^{n} \frac{X_i^2}{\sigma^2}$. Find the distribution of W. (That is, the distribution of W has a name. Name the distribution and give the value of the parameter.) Show your work.

8. (3 points) Again, let $X_1, \ldots, X_n$ be a random sample from a Normal $(0,\sigma^2)$ distribution. Derive a $(1-\alpha)100\%$ confidence interval for σ^2. Note that because $\mu=0$ is known, it's not exactly the same as what you might remember. If you use your memory here instead of working out the details you will be lucky to get half credit.

9. Again, let $X_1, \ldots, X_n$ be a random sample from a Normal $(0,\sigma^2)$ distribution. We wish to test H_0: $\sigma=1$ vs H_1: $\sigma\neq1$. We sample 500 observations and observe $\sum_{i=1}^{n} x_i^2 = 638.07$. Perform a large sample likelihood ratio Chi–square test.

a) (8 points) What is the value of the statistic $G = -2\ln(\lambda)$? Show your work.

b) (1 points) What is the critical value at $\alpha=.05$?

c) (1 point) Do you reject H_0? Answer yes or no.

d) (1 point) Do these results support the idea that $\sigma=1$? Answer yes or no.

10. (5 points) A bowl contains seven marbles of which θ are blue. Thus, the unknown parameter θ is an integer from zero to 7. In order to test the null hypothesis $\theta=2$ against the alternative $\theta=4$, two of the marbles are randomly drawn without replacement and the null hypothesis is rejected if and only if both are red. What is the probability of a type I error?

3. (25 points) Let $f(x_i;\theta) = \frac{\theta^5}{25} e^{-\theta x_i} x_i^4 I_{\{x_i>0\}}$, where the parameter $\theta>0$. Find the Maximum Likelihood Estimate of θ; don't forget to show that the likelihood function is concave down at $\hat{\theta}$.

4. (25 points) Let $X_1, \ldots, X_n$ be a random sample from a Normal (μ,σ^2) population. Derive a $(1-\alpha)100\%$ confidence interval for μ. Derive an <u>exact</u> confidence interval; that is, do <u>not</u> use the central limit theorem.

5. (10 points) Measurements of the diameters of a random sample of 200 ball bearings made by a certain machine during one week showed a mean of .824 inches and a standard deviation of .042 inches. Give a 99% confidence interval for the true mean diameter of all the ball bearings. There is no need to derive anything, but show some calculations.

5. (10 points) Measurements of the diameters of a random sample of 200 ball bearings made by a certain machine during one week showed a mean of .824 inches and a standard deviation of .042 inches. Give a 99% confidence interval for the true mean diameter of all the ball bearings. There is no need to derive anything, but show some calculations.

1. Let $X_1, \ldots X_n \stackrel{\text{i.i.d.}}{\sim}$ Normal(μ,σ^2). Write the likelihood function $L[(\mu,\sigma),\mathbf{x}]$. Don't leave it in terms of $\prod_{i=1}^{n}$ -- carry out the multiplication.

2. Let $X_1, \ldots X_n \stackrel{\text{i.i.d.}}{\sim}$ Binomial(k,p). Write the likelihood function $L[p,\mathbf{x}]$, omitting indicator functions. Don't leave it entirely in terms of $\prod_{i=1}^{n}$ -- carry out the multiplication where possible.

3. Let $X_1, \ldots X_n \stackrel{\text{i.i.d.}}{\sim}$ Poisson(μ). Write the likelihood function $L[\mu,\mathbf{x}]$, omitting indicator functions. Don't leave it entirely in terms of $\prod_{i=1}^{n}$ -- carry out the multiplication where possible.

4. Let $X_1, \ldots X_n \stackrel{\text{i.i.d.}}{\sim}$ Exponential(θ). Write the likelihood function $L[\theta,\mathbf{x}]$, omitting indicator functions. Don't leave it in terms of $\prod_{i=1}^{n}$; carry out the multiplication.

5. Let $X_1, \ldots X_n \overset{\text{i.i.d.}}{\sim}$ Gamma(α,β). Write the likelihood function $L[(\alpha,\beta),\mathbf{x}]$, omitting indicator functions. Don't leave it entirely in terms of $\prod_{i=1}^{n}$ -- carry out the multiplication where possible.

6. Let $X_1, \ldots X_n \overset{\text{i.i.d.}}{\sim}$ Chi-square(r). Write the likelihood function $L[r,\mathbf{x}]$, omitting indicator functions. Don't leave it entirely in terms of $\prod_{i=1}^{n}$ -- carry out the multiplication where possible.

7. Let $X_1, \ldots X_n \overset{\text{i.i.d.}}{\sim}$ Normal(μ,σ^2). What is the distribution of $\sum_{i=1}^{n} X_i$?

8. Let $X_1, \ldots X_n \overset{\text{i.i.d.}}{\sim}$ Binomial(k,p). What is the distribution of $\sum_{i=1}^{n} X_i$?

9. Let $X_1, \ldots X_n \overset{\text{i.i.d.}}{\sim}$ Poisson(μ). What is the distribution of $\sum_{i=1}^{n} X_i$?

10. Let $X_1, \ldots X_n \overset{\text{i.i.d.}}{\sim}$ Exponential(θ). What is the distribution of $\sum_{i=1}^{n} X_i$?

11. Let $X_1, \ldots X_n \overset{\text{i.i.d.}}{\sim}$ Gamma(α, β). What is the distribution of $\sum_{i=1}^{n} X_i$?

12. Let $X_1, \ldots X_n \overset{\text{i.i.d.}}{\sim}$ Chi-square(r). What is the distribution of $\sum_{i=1}^{n} X_i$?

In questions 13-18, write the letter beside the definition. Letters may repeat. **Questions 13-18 are worth 2 points each.**

A.	Simple hypothesis	I.	Null Hypothesis
B.	Composite hypothesis	J.	Alternative hypothesis
C.	Test of a statistical hypothesis	K.	p-value
D.	Critical Region	L.	Type I error
E.	Power function of a test	M.	Type II error
F.	The power of a test	N.	Significance level
G.	Test statistic		
H.	Best test of a simple vs. a simple hypothesis.		

13. _____ An assertion about the distribution of one or more random variables; this assertion does not completely specify the distribution.

14. _____ A subset of the sample space such that, if the data values fall into this space, H_0 is rejected.

15. ______ The probability of rejecting the null hypothesis when the null hypothesis is false.

16. ______ The maximum probability of rejecting the null hypothesis when the null hypothesis is true.

17. ______ The probability of rejecting the null hypothesis, expressed as a function of the parameter θ.

18. ______ An assertion about the distribution of one or more random variables; this assertion completely specifies the distribution.

19. [8 pts] Let $X_1, \ldots X_n$ be a random sample from an exponential distribution; that is $f(x_1)= \frac{1}{\theta} e^{-x_1/\theta} I_{\{x_1>0\}}$. To test H_0: $\theta=1$ vs H_1: $\theta>1$, we employ the critical region $C=\{ (x_1, \ldots, x_{10}): \bar{x} > 2\}$. Find the power function of this test; you may leave the power function in the form of an integral.

1. Let $X_1, \ldots, X_{100}$ be drawn randomly from a distribution with mean 2 and variance 4. Approximately, what is $P\{\bar{X}<1.9\}$?

a) .971

b) .309 *

c) .691

d) .029

e) 0, because the normal distribution is continuous.

2. Let $X_1, \ldots, X_{49}$ be a random sample from a distribution with density $f(x_i) = I_{\{0<x_i<1\}}$. Approximately, what is $P\{\overline{X}\leq .6\}$?

a) .579

b) .992 *

c) .6

d) .029

3. Suppose the lifetime of a certain kind of light bulb has an exponential distribution with parameter $\theta=1$ month. If you order a box of 100 light bulbs, what is the probability that their average lifetime will be less than one week?

a) .579

b) .992

c) .6

d) .029

e) Approximately zero *

4. Let $X_1=3$, $X_2=4$, $X_3=5$ be a random sample from a Poisson distribution with unknown parameter μ. What is the maximum likelihood estimate of μ?

a) 4 *

b) 3

c) 5

d) 1.63

e) Likelihood function has no real maximum.

5. Let $X_1, \ldots, X_n$ be a random sample from a distribution with density $f(x_i | \theta) = \theta x_i^{\theta-1} I_{\{0<x_i<1\}}$. What is the maximum likelihood estimate of θ?

a) $\overline{X}$

b) S^2

c) $\text{Max}\,\{x_i\}$

d) $\dfrac{n}{-\sum_{i=1}^{n} \log x_i}$ *

e) $\cosh(\sum_{i=1}^{n} \log x_i)$

6. Let $X_1=0$, $X_2=1$, $X_3=0$, $X_4= 0$ be a random sample from a Bernoulli [Binomial(1,p)] distribution with unknown parameter p. What is the maximum likelihood estimate of p?

a) 0

b) .25 *

c) .50

d) .75

e) $\text{Max}\,\{x_i\} = 1.0$

7. Let $X_1, \ldots, X_{40}$ be a random sample from a normal distribution with unknown mean μ and **known** variance $\sigma^2=10$. Given a sample mean of $\overline{X} = 7.164$, find a 90% confidence interval for μ.

a) (6.52, 7.81)

b) (6.18, 8.14)

c) (4.06, 10.26)

d) (6.34, 7.99) *

e) (4.564, 7.964)

8. A random sample of 16 fish is selected from the hold of a fishing vessel. The mean length is $\overline{X}$=37 cm., with a sample standard deviation of S=3.873. Assuming that the lengths are from a normal distribution, use the results of question 13 to find a 99% confidence interval for the unknown population mean length μ.

a) (35.247, 38.753)

b) (34.43, 39.58)

c) (34.053, 39.947) *

d) (34.389, 39.602)

e) (34.67, 39.33)

9. Let X_1, ..., X_{169} be a random sample from a population with finite but unknown variance. If $\overline{X}$ = 5 and the sample standard deviation S=13, find a 90% confidence interval for the unknown population mean μ.

a) (3.04, 6.96)

b) (3.355, 6.645) *

c) (2.91762, 4.40124)

d) (3.718, 6.282)

e) (.11929, 14.2213)

10. A public opinion survey of 1000 Canadian adults found that 140 favored a certain political party. Find a 95% confidence interval for the unknown population percentage of Canadian adults who favor the party.

a) (12.19, 15.81)

b) (13.3, 14.7)

c) (13.4, 14.6)

d) (11.85, 16.15) *

e) (-.1546, +1.645)

11. (10 points) Let $X_1, \ldots, X_n$ be a random sample from a distribution with density $f(x_i | \theta) = \theta e^{-\theta x_i} I_{\{x_i > 0\}}$. Find the MLE for θ; to save time, <u>don't bother</u> to show it's a maximum rather than a minimum or saddle point.

12. Let $Y_n \sim \text{Binomial}(n, p_n)$, where $p_n = \mu/n$; μ is a constant. Using limiting moment-generating functions, show $Y_n \xrightarrow{d} Y$, where $Y \sim \text{Poisson}(\mu)$.

13. (20 points) Let $X_1, \ldots, X_n$ be a random sample from a $N(\mu,\sigma^2)$ distribution. Using the following facts, derive a $(1-\alpha)100\%$ confidence interval for μ.

a) If $W \sim N(0,1)$ and $V \sim \chi^2(r)$ are independent, then $T = \dfrac{W}{\sqrt{V/r}} \sim t(r)$.

b) If $X_1, \ldots, X_n \sim$ i.i.d. $N(\mu,\sigma^2)$, then $\overline{X} \sim N(\mu,\sigma^2/n)$, $\dfrac{nS^2}{\sigma^2} \sim \chi^2($nÐ), and S^2 & $\overline{X}$ are independent.

14. (10 points) Let $X_1, \ldots, X_n$ be a random sample from a distribution with mean μ and variance σ^2. Show that $\dfrac{n}{n-1} S^2$ is an unbiased estimate of σ^2.

1. (10 pts) Let $X_1, \ldots, X_n$ be a random sample from a $N(\mu,\sigma^2)$ population. Derive a (1Ðα)100% confidence interval for σ^2.

2. (5 pts) A random sample of size $n=15$ from a $N(\mu,\sigma^2)$ population yields $\overline{x} = 3.2$ and $s^2 = 4.24$. Give a 90% confidence interval for μ. That is, give an interval of the form (a,b). You are being asked for the <u>numbers</u> a & b; you don't have to derive the form of the confidence interval.

5. Ninety consumers each tasted three brands of beer without knowing the brand, and indicated which one they liked best. Thirty-two preferred brand A, 30 preferred brand B and 28 preferred brand C. The company is interested in whether consumers differ in their preferences for these brands at the α=.05 level of significance. Answer the questions below.

a) (2 pts) Under the null hypothesis of equal preferences, what are the expected frequencies?

b) (2 pts) What is the value of the test statistic?

c) (2 pts) Under the null hypothesis, what is the (approximate) distribution of the test statistic?

d) (2 pts) What is the critical value for the test statistic at α=.05?

e) (2 pts) Do you reject H_0?

f) (2 pts) Are the results statistically significant?

g) (2 pts) What do you conclude about consumers' preferences for these brands of beer?

6. Let $X_1, \ldots, X_n$ be a random sample from a Poisson distribution with mean μ. To test H_0: $\mu \geq 0.5$ vs H_1: $\mu < 0.5$, we employ the critical region $C = \{(x_1, \ldots, x_n): \sum_{i=1}^{n} x_i = 0\}$.

a) (3 pts) Find the power function of the test.

b) (2 pts) What is the maximum value of this function when H_0 is true?

c) (2 pts) What is the maximum probability of a type I error?

d) (2 pts) What is the significance level α?

7. (15 pts) Let $X_1, \ldots, X_n$ be a random sample from a $N(\mu,\sigma^2)$ distribution; assume that σ^2 is a KNOWN constant.
Use the Neyman-Pearson theorem to show that a best critical region of size α for H_0: $\mu=\mu_0$ vs H_1: $\mu=\mu_1$, where $\mu_1>\mu_0$, may be based on $\overline{x}$.

8. (10 pts) If $X_1, \ldots, X_n$ is a random sample from a distribution with density $f(x;\theta)=\theta x^{\theta-1}$ for $0<x<1$ and zero elsewhere, show that a best critical region of size α for H_0: $\theta=1$ vs H_1: $\theta=2$ is
$\{(x_1, \ldots, x_n): c \leq \prod_{i=1}^{n} x_i\}$.

9. (15 pts) Let $X_1, \ldots, X_n$ be a random sample from a $N(\mu,1)$ distribution. Show that the likelihood ratio test of H_0: $\mu=\mu_0$ vs H_1: $\mu \neq \mu_0$ gives rise to the critical region $\{(x_1, \ldots, x_n): |\overline{x}-\mu_0| \geq c\}$.

1. (5 pts) Define $Y_n \xrightarrow{p} Y$. I'll start it for you. Let Y_1, Y_2, ... be a sequence of random variables, and let Y be another random variable. Y_n is said to converge to Y in probability if (continue)

2. (5 pts) State the Central Limit Theorem. I'll start it for you. Let X_1, ..., X_n be a random sample from a distribution with mean μ and variance σ^2. (continue)

3. (5 pts) State the Weak Law of Large Numbers. I'll start it for you. Let X_1, ..., X_n be a random sample from a distribution with mean μ. (continue)

4. (5 pts) State Chebyshev's inequality. I'll start it for you. Let X be a random variable with mean μ and variance σ^2, and let k be any positive number. (continue)

5. (5 pts) Define "$\widehat{\Theta}_n$ is consistent for θ." I'll start it for you. Let X_1, ..., X_n be a random sample from a distribution with parameter θ, and let $\widehat{\Theta}_n = \widehat{\Theta}_n(X_1, ..., X_n)$ be a statistic. $\widehat{\Theta}_n$ is said to be consistent for θ if (continue)

6. (5 pts) Define "$\widehat{\Theta}_n$ is unbiased for θ." I'll start it for you. Let X_1, ..., X_n be a random sample from a distribution with parameter θ, and let $\widehat{\Theta}_n = \widehat{\Theta}_n(X_1, ..., X_n)$ be a statistic. $\widehat{\Theta}_n$ is said to be unbiased for θ if (continue)

7. (10 pts) A multiple choice test of adult "intelligence" is carefully constructed so that the mean score of English-speaking adults is μ=100, with a standard deviation of σ=10. If 100 adults are randomly chosen from this population, what is the probability that their average score is less than 99?

8. Let $X_1, \ldots, X_n$ be a random sample from a distribution with mean μ and variance σ^2.

a. (1 pt) What is $E(\overline{X}_n)$?

b. (1 pt) What is $Var(\overline{X}_n)$?

c. (8 pts) Use Chebyshev's inequality to prove the Weak Law of Large Numbers.

9. Let $X_1, \ldots, X_n$ be a random sample from a Bernoulli distribution. That is, $f_{X_i}(x_i;p) = p^{x_i}(1-p)^{1-x_i}$ for $x_i = 0$ or 1, and zero otherwise; $0<p<1$.

a. (7 pts) Find the Maximum Likelihood Estimate $\hat{p}$ of p.

b. (2 pts) Show that the likelihood function really has a maximum at $\hat{p}$.

c. (3 pts) Show that $\hat{p}$ is unbiased for p.

d. (3 pts) Show that $\hat{p}$ is consistent for p.

10. Let $X_1, \ldots, X_n$ be a random sample from a distribution with density $f_{X_i}(x_i;\alpha) = e^{-(x_i-\alpha)}$ for $x_i>\alpha$, and zero otherwise; α is an unknown constant greater than zero.

a. (10 pts) Find the Method of Moments estimator $\hat{\alpha}_n$ of α.

b. (2 pts) Show $\hat{\alpha}_n$ is unbiased for α.

c. (3 pts) Show $\hat{\alpha}_n$ is consistent for α.

11. (10 pts) Let $X_1, \ldots, X_n$ be a random sample from a distribution with density $f_{X_i}(x_i;\theta) = \dfrac{e^{-x_i}}{1-e^{-\theta}}$ for $0<x_i<\theta$, and zero otherwise; θ is an unknown constant greater than zero. Find the Maximum Likelihood Estimate of θ.

12. (10 pts) Let G_n have a Gamma distribution with parameters $\alpha=n$ and β, where $\beta>0$ is not a function of n; that is, $f_{G_n}(x;\beta) = \dfrac{1}{\beta^n \Gamma(n)} e^{-x/\beta} x^{n-1}$ for $x\geq 0$, and zero otherwise. Let $Y_n = \dfrac{G_n}{n}$. Find the limiting distribution of Y_n.

2. Let $X_1, \ldots X_k \overset{\text{i.i.d.}}{\sim}$ Binomial(n,p). What is the distribution of $Y = \sum_{i=1}^{n} x_i$?

   a) Binomial(nk,p) *

   b) Binomial($\sum_{i=1}^{n} x_i$,p)

   c) Binomial(n,kp)

   d) Binomial(k,np)

   e) Normal(np,npq)

3. Let $X_1, \ldots X_n \overset{\text{i.i.d.}}{\sim}$ Exponential(θ). What is the <u>exact</u> distribution of $\overline{X} = \frac{1}{n}\sum_{i=1}^{n} x_i$?

   a) Normal ($\mu = \theta$, $\sigma^2 = \dfrac{\theta^2}{n}$)

   b) Exponential ($n\theta$)

   c) Normal ($\mu = \theta$, $\sigma^2 = \dfrac{\theta}{n}$)

d) Gamma ($\alpha = n$, $\beta = \frac{\theta}{n}$) *

e) Gamma($\alpha = n$, $\beta = \theta$)

4. Let $X_1, \ldots X_n$ be independent $\chi^2(r_i)$ random variables. What is the <u>exact</u> distribution of $Y = \sum_{i=1}^{n} x_i$?

a) Gamma(α=2, $\beta=.5\sum_{i=1}^{n} r_i$)

b) Normal($\mu=\overline{X}$, $\sigma^2=S^2$)

c) $\chi^2(nr_i)$

d) $\chi^2(\sum_{i=1}^{n} r_i)$ *

e) $\chi^2(n-1)$

5. Let $X_1, \ldots X_n \overset{i.i.d.}{\sim} N(\mu,\sigma^2)$. What is the limiting distribution of the sample mean $\overline{X}_n$?

a) Normal (0,1)

b) Limiting moment generating function is multivariate.

c) Degenerate at μ

d) Degenerate at 0 *

e) Normal (μ,σ^2)

6. Let Y_n have distribution function $F_n(y) = [1 - (1-\frac{y}{n})^n]\ I_{\{y>0\}}$. What is the limiting distribution of Y_n?

a) Exponential (θ=1) *

b) Normal ($\mu=0,\sigma^2=1$)

c) Poisson (μ=1)

d) Binomial (1,.5)

e) t(r=1)

7. Suppose that the average hand gripping strength of Canadian Industrial workers is 110 pounds, with a variance of 100. For a sample of 75 workers, what is the probability that their mean gripping strength will be greater than 111 pounds?

a)

b) .193 *

c)

d)

e)

8. Let $X_1=1$, $X_2=3$, $X_3=5$ be a random sample from a normal distribution with unknown mean μ and unknown variance σ^2. What is the method of moments estimator for μ?

a) 1

b) 3 *

c) 5

d) 9

e) $S^2/\overline{X}$

9. Let $X_1=2$, $X_2=9$, $X_3=1$ be a random sample from a distribution with density $f(x_i) = \frac{e^{-\lambda}\lambda^{x_i}}{x_i!} I_{\{x_i=0,\ 1,\ \ldots\}}$. Find the maximum likelihood estimate of λ.

a) 0

b) 2

c) 4 *

d) 12

e) $e \approx 2.172828$

10. Let $X_1, \ldots X_n$ be a random sample from a distribution with density $f(x_i) = \lambda e^{-\lambda x_i} I_{\{x_i > 0\}}$. What is the method of moments estimator for λ?

a) $\overline{X}_n$

b) $\frac{1}{n}\sum_{i=1}^{n} x_i^2$

c) $\frac{1}{n}\sum_{i=1}^{n} x_i^2 - \overline{X}^2$

d) $n/\sum_{i=1}^{n} x_i^2$

e) $1/\overline{X}_n$ *

11. A public opinion survey of 1000 Canadian adults found that 20% agreed with the statement "Abortion should be illegal except when the mother's life is in danger". Find a 95% confidence interval for the unknown population percentage of Canadian adults who actually agree with the statement.

a) (18.19, 21.81)

b) (17.52, 22.48) *

c) (19.4, 20.6)

d) (17.85, 22.15)

e) (-1.96, +1.96)

12. A single observation X_1 is sampled from a distribution with density $f(x_i) = \frac{1}{\theta} e^{-x_i/\theta} I_{\{x_i>0\}}$. To test H_0: $\theta=1$, reject H_0 if $X_1>3$. What is the power of this test at $\theta=5$?

a) .4512

b) .5488 *

c) .1889

d) Approximately zero

e) .8111

13. Let $X_1, \ldots X_9 \overset{\text{i.i.d.}}{\sim} N(\mu,1)$. To test the simple null hypothesis of $\mu=0$ against the composite alternative of $\mu>0$, a critical region of $C=\{(x_1, \ldots, x_n): \bar{x}_n>0.6\}$ is employed. What is the power of this test if the actual value of μ is 1?

a) .274

b) .726

c) .115

d) .885 *

e) Power is not defined in this situation

14. Let $X_1, X_2 \overset{\text{i.i.d.}}{\sim}$ Poisson(μ). To test the simple null hypothesis of $\mu=1$ against the composite alternative of $\mu>0$, a critical region of $C=\{(x_1, \ldots, x_n): \sum_{i=1}^{n} X_i>3\}$ is employed. What is the significance level of this test?

a) α = .143 *

b) α = .2707

c) α = .947

d) α = .0613

e) α = .05

In questions 15-26 below, write the letter beside the definition. Letters may repeat. **Questions 15-26 are worth one point each.**

A.	Simple hypothesis	H.	Null Hypothesis
B.	Composite hypothesis	I.	Alternative hypothesis
C.	Test of a statistical hypothesis	J.	p-value
D.	Critical Region	K.	Type I error
E.	Power function of a test	L.	Type II error
F.	The power of a test	M.	Significance level
	Test statistic		
G.	Best test of a simple vs. a simple hypothesis.		

15. ______ The critical region is defined in terms of this function of the sample data.

16. ______ An assertion about the distribution of one or more random variables; this assertion completely specifies the distribution.

17. ______ A subset of the sample space such that, if the data values fall into this space, H_0 is rejected.

18. ______ The probability of rejecting the null hypothesis when the null hypothesis is true.

19. ______ To reject H_0 when H_0 is in fact true.

20. ______ The maximum probability of rejecting the null hypothesis when the null hypothesis is true.

21. ______ The smallest significance level at which the null hypothesis can be rejected.

22. ______ The conjecture that the investigator is interested in supporting, before any sample data are collected.

23. ______ The probability of rejecting the null hypothesis, expressed as a function of the parameter θ.

24. ______ To fail to reject H_0 when H_0 is false.

25. ______ An assertion about the distribution of one or more random variables; this assertion does not completely specify the

distribution.

26. ______ A rule, based upon sample values, that leads to a decision to reject or not reject H_0.

27. (8 points) Let X_1 and X_2 be independent N(0,1) random variables. Let $Y_1=X_1/X_2$ and $Y_2=X_2$. Ignoring the value of Y_1 at $X_2=0$ (after all, this is an event of probability zero), find the joint density of Y_1 and Y_2; don't forget the absolute value of the Jacobian.

29. (?? points) Let $X_1, \ldots X_n$ be a random sample from a distribution with density $f(x_i) = \frac{1}{\theta} I_{\{0<x_i<\theta\}}$.

a) Find the method of moments estimator for θ.

b) Show that this estimator is consistent.

30. (8 points) Let $X_1, \ldots X_n$ be a random sample from a distribution with density $f(x_i) = \frac{e^{-\lambda}\lambda^{x_i}}{x_i!} I_{\{x_i=0,\ 1,\ \ldots\}}$. Show that $\sum_{i=1}^{n} X_i$ is sufficient for λ.

31. (4 points) Let $X_1, \ldots X_{16}$ be a random sample from a $N(\mu,1)$ distribution.

a) Using the Neyman-Pearson theorem, show that the Unifomly Most Powerful test for H_0: $\mu=\mu_0$ versus H_1: $\mu>\mu_0$ may be based on the value of the sample mean.

32. (4 points) Let $X_1, \ldots X_n$ be a random sample from a $N(\mu,\sigma^2)$ distribution where σ^2 is known. Show that the likelihood ratio test for H_0: $\mu=0$ versus H_1: $\mu\neq 0$ may be based on the absolute value of the sample mean.

33. (4 points) Let (X_1,Y_1), (X_2,Y_2), ..., (X_n,Y_n) be a sample of bivariate data. The equation of the least-squares line through the origin is $Y=bX$. Find the expression for the slope b.

34. (12 points) The reaction times of 26 students are measured before and after drinking 6 ounces of grain alcohol. There is reason to believe that the scores X_i= After minus Before are normally distributed. Because the data can be assumed normal, the sample size is small and the true population variance is unknown, a scientist decides to use a one-sample t-test to determine whether the treatment had any effect. The scientist observes a sample standard deviation of S=0.1 and a mean of $\overline{X}_n = -0.4$ (Notice, <u>Minus</u> 0.4).

a) What is the model for the data; that is, what are the necessary assumptions for the one-sample t-test?

b) The simple null hypothesis of no treatment effect is

H_0:

c) For a two-tailed test, the alternative hypothesis is

H_1:

d) For a one-tailed test, the most reasonable alternative hypothesis is H_1:

e) At a significance level of α=.05 for a one-tailed test, what is the critical value of the test statistic?

f) At a significance level of α=.01 for a two-tailed test, what is the critical value of the test statistic?

g) At a significance level of α=.05 for a one-tailed test, is H_0 rejected?

h) At a significance level of α=.01 for a two-tailed test, is H_0 rejected?

i) At α=.05 for a one-tailed test, is μ significantly different from zero?

j) At α=.01 for a two-tailed test, is μ significantly different from zero?

k) For the two-tailed test, what conclusions can you draw about alcohol consumption and reaction time?

1. (3 pts) In a certain city, the average birth weight of babies is known to be six and one-half pounds, with a standard deviation of one pound. If we select a random sample of 100 babies, what is the probability that their mean birth weight will be less than 6.25 pounds?

2. Let the joint density of the continuous random variables X_1 and X_2 be $f_{X_1X_2}(x_1,x_2) = 12x_1x_2(1-x_2)I_{\{0<x_1<1\}}I_{\{0<x_2<1\}}$; let $Y_1=X_1X_2^2$ and $Y_2=X_2$.

a. (6 pts) Find the joint density of Y_1 and Y_2; be sure to indicate the support.

b. (2 pts) Draw the support of $f_{Y_1Y_2}(y_1,y_2)$.

c. (2 pts) For what values of y_1 will the marginal density $f_{Y_1}(y_1)$ be non-zero?

d. (4 pts) For a (general) point y_1 in your answer to (c), write the correct limits of integration on the integral representing $f_{Y_1}(y_1)$ below. <u>Don't</u> evaluate the integral. You will get points <u>only</u> for having the correct limits of integration.

$$f_{Y_1}(y_1) = \int f_{Y_1Y_2}(y_1,y_2)\ dy_2$$

3. (6 pts) Let $X \sim N(0,1)$ and $Y=X^2$. Show that $Y \sim \chi^2(1)$. Hint: MGF works faster than the way I did the problem in class.

4. (5 pts) Let $X_1, \ldots, X_n$ be a random sample from a continuous distribution with density f_X and distribution function F_X. Let Y be the maximum of $X_1, \ldots, X_n$. Find the density of Y.

5. Let $X_1, \ldots, X_n$ be a random sample from a distribution (not necessarily normal) with mean μ and variance σ^2.

a. (2 pts) Show that the sample mean $\overline{X}_n$ is an unbiased estimate for μ.

b. (2 pts) Show that $Var(\overline{X}_n) = \sigma^2/n$, using the fact that since $X_1, \ldots, X_n$ are independent, $Var(\sum_{i=1}^{n} X_i)=\sum_{i=1}^{n} Var(X_i)$.

c. (2 pts) Show that $\overline{X}_n$ is consistent for μ.

6. (6 pts) Let $X_1, \ldots, X_n$ be a random sample from a distribution with density $f(x;\theta) = e^{-(x-\theta)} I_{\{x>\theta\}}$, where $\theta>0$ is the unknown parameter. θ. Find the Method of Moments estimator (**NOT** the MLE!) for θ.

7. (10 pts) Let $X_1, \ldots, X_n$ be a random sample from a binomial distribution with parameters k and θ, i.e., X_i has probability function $f(x_i;\theta) = \binom{k}{x_i} \theta^{x_i} (1-\theta)^{k-x_i} I_{\{x_i=0, \ldots, k\}}$; k is known. Find the maximum likelihood estimator for θ.

8. (10 pts) Let $X_1, \ldots, X_9$ be a random sample from a Normal population with known variance $\sigma^2=1$ and unknown mean μ. Use the Neyman-Pearson lemma to show that the most powerful critical region of size $\alpha=.05$ for testing H_0: $\mu=2$ versus H_1: $\mu>2$ is given by $\{(x_1, \ldots, x_n): \overline{x} > 2.5483\}$.

9. Let $X_1, \ldots X_n$ be a random sample from a $N(\mu,\sigma^2)$ distribution where σ^2 is unknown. Given that the MLE of μ is $\overline{X}$, the MLE of σ^2 is $\sum_{i=1}^{n} (X_i-\overline{X})^2/n$, and letting $T=\overline{X}\sqrt{n-1}/S$,

a. (10 pts) Show that the likelihood ratio test for H_0: $\mu=0$ versus H_1: $\mu\neq 0$ may be based on the absolute value of T.

b. (2 pts) Under H_0, what is the distribution of T? (Just give the answer; you don't need to prove anything.) ________________

c. (2 pts) If n=5 and α=.05, what is the critical value of T?

____

d. If $\overline{X}$ = -3.0 and S^2 = 4.0,

i) (1 pt) Do you reject H_0? (yes or no) ________

ii) (1 pt) Are the results statistically significant?
(yes or no) _________

10. (3 pts) A public opinion survey of 1000 Canadian adults found that 20% agreed with the statement "Abortion should be illegal except when the mother's life is in danger". Find a 95% confidence interval for the unknown population percentage of Canadian adults who actually agree with the statement. Don't bother to correct for continuity.

11. Eighteen white rats are assigned at random to either an experimental group or a control group. The experimental group is injected with anabolic steroids and the control group is given a sham injection of a saline solution. Each rat is placed in a cage with a hungry boa constrictor, and its survival time is recorded. Let $X_1, \ldots, X_{n_1}$ denote the survival times of the experimental group and $Y_1, \ldots, Y_{n_2}$ denote the survival times of the control group. Adopting the questionable assumption that $X_1, \ldots, X_{n_1}$ are a random sample from a Normal (μ_1,σ^2) population and that $Y_1, \ldots, Y_{n_2}$ are an independent random sample from a Normal (μ_2,σ^2) population, we seek to test whether the steroid treatment was effective.

a. (2 pts) What is the null hypothesis? ___________________

b. (2 pts) For a one-tailed test, two alternative hypotheses are possible. State the more reasonable one. ____________________

c. (2 pts) State the alternative hypothesis for a two-tailed test. ________________________

d. (2 pts) What is the distribution of the test statistic under the null hypothesis? __________________________

e. (2 pts) If α = .05, what is the critical value of the test statistic for a two-tailed test? ______________________

f. (1 pt) If the value of the test statistic is 2.00, do you reject H_0? (Yes or No) __________

12. In a "twelve ounce" can of beer, the actual amount in the can is a normal random variable with mean μ =12 and and a variance σ^2

that is less than 0.1 ounces if the machinery is working properly. If the quality assurance department can reject H_0: σ^2=0.1 at $\propto$=.01, the machinery must be adjusted.

A random sample of 4 cans yields measurements of 12.0, 11.4, 12.6 and 8.0. Answer the questions below.

a. (2 pts) What is the alternative hypothesis? __________

b. (1 pt) Do we have a one-tailed test, or two? ________

c. (2 pts) What is the numerical value of the test statistic? (Calculate this from the sample data.)

Answ to (c) ________

d. (2 pts) What is the critical value of the test statistic?

____

e. (1 pt) Do you reject H_0 at $\propto$=.01? (Yes or no) _______

f. (1 pt) Are the results statistically significant? (Yes or no) ___________

g. (1 pt) Must the machinery be adjusted? (Yes or no)

____