STA 261 Formulas

Distribution	$f(x)$	$M(t)$	$E(X)$	$Var(X)$
Bernoulli	$\theta^x(1-\theta)^{1-x}I(x=0,1)$	$\theta e^t+1-\theta$	θ	$\theta(1-\theta)$
Binomial	$\binom{n}{x}\theta^x(1-\theta)^{n-x}I(x=0,\dots,n)$	$(\theta e^t+1-\theta)^n$	$n\theta$	$n\theta(1-\theta)$
Poisson	$\frac{e^{-\lambda}\lambda^x}{x!}I(x=0,1,\dots)$	$e^{\lambda(e^t-1)}$	λ	λ
Geometric	$\theta(1-\theta)^{x-1}I(x=1,2,\dots)$	$\theta(e^{-t}+\theta-1)^{-1}$	$\frac{1}{\theta}$	$\frac{1-\theta}{\theta^2}$
Uniform	$\frac{1}{\beta-\alpha}I(\alpha\le x\le\beta)$	$\frac{e^{\beta t}-e^{\alpha t}}{t(\beta-\alpha)}$	$\frac{\alpha+\beta}{2}$	$\frac{(\beta-\alpha)^2}{12}$
Exponential	$\frac{1}{\theta}e^{-x/\theta}I(x>0)$	$(1-\theta t)^{-1}$	θ	θ^2
Gamma	$\frac{1}{\beta^\alpha\Gamma(\alpha)}e^{-x/\beta}x^{\alpha-1}I(x>0)$	$(1-\beta t)^{-\alpha}$	$\alpha\beta$	$\alpha\beta^2$
Chi-square	$\frac{1}{2^{\nu/2}\Gamma(\nu/2)}e^{-x/2}x^{\nu/2-1}I(x>0)$	$(1-2t)^{-\nu/2}$	ν	2ν
Normal	$\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$	$e^{\mu t+\frac{1}{2}\sigma^2t^2}$	μ	σ^2

$T_n \xrightarrow{P} T$ means for every $c>0$, $\lim_{n\to\infty} P\{|T_n-T|<c\}=1$

$T_n \xrightarrow{d} T$ means for every continuity point t of F_T, $\lim_{n\to\infty} F_{T_n}(t)=F_T(t)$.

For a random sample from a normal distribution, $\frac{(n-1)S^2}{\sigma^2}\sim\chi^2(n-1)$.

If $Z\sim N(0,1)$ and $Y\sim\chi^2(\nu)$ are independent, then $T=\frac{Z}{\sqrt{Y/\nu}}\sim t(\nu)$.

If $U\sim\chi^2(\nu_1)$ and $V\sim\chi^2(\nu_2)$ are independent, then $F=\frac{U/\nu_1}{V/\nu_2}\sim F(\nu_1,\nu_2)$.

$$e^x=\sum_{k=0}^{\infty}\frac{x^k}{k!}$$

$$\sum_{k=r}^{\infty}a^k=\frac{a^r}{1-a} \text{ for } 0<a<1$$

$$E(g(X))\ge aPr\{g(X)\ge a\} \text{ for } g(x)\ge 0$$

$$Pr\{|X-\mu|\ge k\sigma\}\le\frac{1}{k^2}$$

$$F_{Y_1}(y)=1-[1-F(y)]^n$$

$$f_{Y_1}(y)=n[1-F(y)]^{n-1}f(y)$$

$$F_{Y_n}(y)=[F(y)]^n$$

$$f_{Y_n}(y)=n[F(y)]^{n-1}f(y)$$

$$V(\widehat{\Theta})\ge\frac{1}{nE\left([\frac{\partial}{\partial\theta}\ln f(X;\theta)]^2\right)}$$

$$S^2=\frac{\sum_{i=1}^n(X_i-\overline{X})^2}{n-1}=\frac{1}{n-1}\left(\sum_{i=1}^n X_i^2-n\overline{X}^2\right)$$

$$Z_n=\frac{\sqrt{n}(\overline{X}_n-\mu)}{\widehat{\sigma}}\xrightarrow{d} Z\sim N(0,1)$$

$$1-\alpha\approx P\{\overline{X}_n-z_{\alpha/2}\frac{\widehat{\sigma}}{\sqrt{n}}\le\mu\le\overline{X}_n+z_{\alpha/2}\frac{\widehat{\sigma}}{\sqrt{n}}\}$$

$$Z=\frac{(\overline{X}_{n_1}-\overline{Y}_{n_2})-(\mu_1-\mu_2)}{\sqrt{\frac{\widehat{\sigma}_1^2}{n_1}+\frac{\widehat{\sigma}_2^2}{n_2}}}$$

$$\overline{X}_{n_1}-\overline{Y}_{n_2}\pm z_{\alpha/2}\sqrt{\frac{\widehat{\sigma}_1^2}{n_1}+\frac{\widehat{\sigma}_2^2}{n_2}}$$

$$T=\frac{(\overline{X}-\overline{Y})-(\mu_1-\mu_2)}{S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}\sim t(n_1+n_2-2), \text{ where } S_p=\sqrt{\frac{(n_1-1)S_1^2+(n_2-1)S_2^2}{n_1+n_2-2}}$$

$$1-\alpha=P\left\{\frac{S_1^2}{S_2^2}\frac{1}{f_{\alpha/2,n_1-1,n_2-1}}<\frac{\sigma_1^2}{\sigma_2^2}<\frac{S_1^2}{S_2^2}f_{\alpha/2,n_2-1,n_1-1}\right\}$$

$$\lambda=\frac{\max_{\theta\in\Theta_0}L(\theta,\mathbf{x})}{\max_{\theta\in\Theta}L(\theta,\mathbf{x})}=\frac{L(\widetilde{\theta},\mathbf{x})}{L(\widehat{\theta},\mathbf{x})}$$