ABOUT TEST 2

Tues March 10
IB 120 11:30 - 12:30

Assignments 4 - 6

Lecture Sections 4 - 6
Thurs Jan 23 - Tues March 3

Formula sheet posted

7 Q, One Q per page, abc

Find prove
Estimation followed with data
Test followed with data

Remember dist facts, vocab

Questions use vocabulary

Bring a calculator

If ask what's chemotherapy, OKAY.

Ask what's a critical value, no answer.

Thursday March 5

Have a good collection of tests

- $X_1, \dots X_n \overset{iid}{\sim}$ Bernoulli (θ) $\quad H_0: \theta = \theta_0$

$$Z = \frac{\sqrt{n}(\bar{X}_n - \theta_0)}{\sqrt{\theta_0(1-\theta_0)}} \qquad \begin{matrix} \theta \le \theta_0 \\ \theta \ge \theta_0 \end{matrix}$$

- $X_1 \dots X_n \overset{iid}{\sim} ?(\mu, \sigma^2)$ $\quad H_0: \mu = \mu_0$ vs $\mu \ne \mu_0$

$$Z = \frac{\sqrt{n}(\bar{X}_n - \theta_0)}{S_n} \qquad \begin{matrix} \mu \le \mu_0 \text{ vs } \mu > \mu_0 \\ \mu \ge \mu_0 \text{ vs } \mu < \mu_0 \end{matrix}$$

- $X_1 \dots X_n \overset{iid}{\sim} N(\mu, \sigma^2)$ $\quad H_0: \mu = \mu_0$

$$T = \frac{\sqrt{n}(\bar{X}_n - \mu_0)}{S_n} \sim t(n-1) \qquad \begin{matrix} \mu \le \mu_0 \\ \mu \ge \mu_0 \end{matrix}$$

$$H_0: \sigma^2 = \sigma_0^2 \qquad \begin{matrix} \sigma^2 \le \sigma_0^2 \\ \sigma^2 \ge \sigma_0^2 \end{matrix} \qquad Y = \frac{(n-1)S^2}{\sigma_0^2} \overset{H_0}{\sim} \chi^2(n-1)$$

Extend to 2 independent normal samples

$H_0: \mu_1 = \mu_2, \ \mu_1 \le \mu_2, \ \mu_1 \ge \mu_2$ T-test

$H_0: \sigma_1^2 = \sigma_2^2 \quad \le \quad \ge$ F-test

Testing several group means (treatments)
Like Drug ABC Cookie receipe 1 2 3 4
Have 2-sample T-Test ; Extend

$$T = \frac{Z}{\sqrt{W/\nu}} , \quad T^2 = \frac{Z^2/1}{W/\nu} \sim F(1, \nu)$$

Generalize F-test

<u>Model</u> $x_{ij} \overset{ind}{\sim} N(\mu_j, \sigma^2)$ $\begin{matrix} j = 1, \ldots, k \text{ Groups} \\ i = 1, \ldots, n_j \end{matrix}$

k groups, n_j in group j, $j = 1, \ldots, k$
$n = \sum_{j=1}^{k} n_j$

$H_0 : \mu_1 = \mu_2 = \cdots = \mu_k$ vs H_1 : Not all equal

Want F : Denominator is natural

$$F = \frac{W_1/\nu_1}{W_2/\nu_2}$$

$$W_2 = \frac{(n_1 - 1)S_1^2}{\sigma^2} + \frac{(n_2 - 1)S_2^2}{\sigma^2} + \cdots + \frac{(n_k - 1)S_k^2}{\sigma^2}$$

$\sim \chi^2(n-k)$ whether H_0 is true or not

$$W_2 = \sum_{j=1}^{k} \frac{(n_j-1)S_j^2}{\sigma^2} = \frac{\sum_{j=1}^{k}\sum_{i=1}^{n_j}(X_{ij}-\bar{X}_j)^2}{\sigma^2}$$ 42

Sum of squares <u>within</u> groups

For numerator, T^2 has $(\bar{X}-\bar{Y})^2$. Replace with another sum of squares.

$$T = \frac{\bar{X}-\bar{Y}-0}{S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$$

Natural idea: $\sum_{j=1}^{k}(\bar{X}_j - \bar{\bar{X}})^2$

Mean of means

Equal sample sizes: Too restrictive

Instead, measure squared deviation around

$$\bar{X}_\cdot = \sum_{j=1}^{k}\frac{n_j}{n}\bar{X}_j = \frac{1}{n}\sum_{j=1}^{k}\sum_{i=1}^{n_j}X_{ij}$$

overall mean

USE $\sum_{j=1}^{k} n_j(\bar{X}_j - \bar{X}_\cdot)^2$

Familiar trick

$$\sum_{j=1}^{k}\sum_{i=1}^{n_j}(X_{ij}-\bar{X}_\circ)^2 = \sum_{j=1}^{k}\sum_{i=1}^{n_j}(\underbrace{X_{ij}-\bar{X}_j}_{a}+\underbrace{\bar{X}_j-\bar{X}_\circ}_{b})^2$$

$$= \sum_{j=1}^{k}\sum_{i=1}^{n_j}\left[(X_{ij}-\bar{X}_j)^2+2(X_{ij}-\bar{X}_j)(\bar{X}_j-\bar{X}_\circ)+(\bar{X}_j-\bar{X}_\circ)^2\right]$$

$$= \sum_{j=1}^{k}\sum_{i=1}^{n_j}(X_{ij}-\bar{X}_j)^2+2\sum_{j=1}^{k}(\bar{X}_j-\bar{X}_\circ)\sum_{i=1}^{n_j}(X_{ij}-\bar{X}_j)$$

$\sum X_{ij} - n\bar{X}_j = \sum X_{ij} - \sum X_{ij} = 0$

$$+\sum_{j=1}^{k}\sum_{i=1}^{n_j}(\bar{X}_j-\bar{X}_\circ)^2$$

$$= \sum_{j=1}^{k}\sum_{i=1}^{n_j}(X_{ij}-\bar{X}_j)^2+\sum_{j=1}^{k}n_j(\bar{X}_j-\bar{X}_\circ)^2$$

Sum of Squares within Groups

Sum of Squares Between Groups

Have $\quad Y_0 = Y_1 + Y_2$

$$\frac{\sum_{j=1}^{k}\sum_{i=1}^{n_j}(X_{ij}-\bar{X}_\bullet)^2}{\sigma} = \frac{\sum_{j=1}^{k}n_j(\bar{X}_j-\bar{X}_\circ)}{\sigma^2}+\frac{\sum_{j=1}^{k}\sum_{i=1}^{n_j}(X_{ij}-\bar{X}_j)^2}{\sigma^2}$$

DOT

$$\sum_{j=1}^{k}\frac{(n_j-1)S_j^2}{\sigma^2}$$

$Y \sim \chi^2(n-1)$

$Y_1 \sim \chi^2(k-1)$ —

$\chi^2(n-k)$