Name Jerry

Student Number /

STA 260 S2020 Practice Exam 1

This is not a full practice exam, though it is a reasonable question. The goal is to make sure the system works.

1. (100 Points) Let $X_1, \ldots, X_{n_1}$ be a random sample from a distribution with expected value μ and *known* variance σ_1^2. Independently of the X_j, let $Y_1, \ldots, Y_{n_2}$ be a random sample from a distribution with the same expected value μ and known variance σ_2^2. We will estimate μ with $\widehat{\mu} = a\,\overline{X} + (1-a)\,\overline{Y}$, where $0 \leq a \leq 1$.

   (a) Show that $\widehat{\mu}$ is unbiased for μ.

$$E(\hat{\mu}) = E(a\bar{X} + (1-a)\bar{Y}) = aE(\bar{X}) + (1-a)E(\bar{Y})$$

$$= a\mu + (1-a)\mu = \mu(a+1-a) = \mu \quad \text{unbiased}$$

   (b) Find the value of a that makes $Var(\widehat{\mu})$ as small as possible. Show your work. **Circle your final answer**.

$$v = Var(\hat{\mu}) = Var(a\bar{X} + (1-a)\bar{Y}) \overset{ind}{=} a^2 Var(\bar{X}) + (1-a)^2 Var(\bar{Y})$$

$$= a^2 \frac{\sigma_1^2}{n_1} + (1-a)^2 \frac{\sigma_2^2}{n_2}$$

$$\frac{dv}{da} = 2a\frac{\sigma_1^2}{n_1} + 2(1-a)(-1)\frac{\sigma_2^2}{n_2} = 2a\frac{\sigma_1^2}{n_1} - 2(1-a)\frac{\sigma_2^2}{n_2} \overset{set}{=} 0$$

$$\Rightarrow a\frac{\sigma_1^2}{n_1} = (1-a)\frac{\sigma_2^2}{n_2} = \frac{\sigma_2^2}{n_2} - a\frac{\sigma_2^2}{n_2} \Rightarrow a\left(\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}\right) = \frac{\sigma_2^2}{n_2}$$

$$\Rightarrow a = \frac{\sigma_2^2/n_2}{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

Checking that it's a minimum,

$$\frac{d^2v}{da^2} = \frac{d}{da}\left(2a\frac{\sigma_1^2}{n_1} - 2\frac{\sigma_2^2}{n_2} + 2a\frac{\sigma_2^2}{n_2}\right)$$

$$= 2\left(\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}\right) > 0 \quad \text{ccup} \cup$$

MIN