Assignment 8

(1) For $k=2$, $F = \dfrac{SSB/(k-1)}{SSW/(n-k)} = \dfrac{SSB}{SSW/(n_1+n_2-2)}$

$$SSW = \sum_{i=1}^{n_1} (X_{i1} - \bar{X}_1)^2 + \sum_{i=1}^{n_2} (X_{i2} - \bar{X}_2)^2$$
$$= (n_1-1)S_1^2 + (n_2-1)S_2^2, \text{ so}$$

$$\frac{SSW}{n_1+n_2-2} = S_p^2$$

$$SSB = n_1(\bar{X}_1 - \bar{X}_.)^2 + n_2(\bar{X}_2 - \bar{X}_.)^2$$

Now, $\bar{X}_1 - \bar{X}_. = \bar{X}_1 - \left(\frac{n_1}{n}\bar{X}_1 + \frac{n_2}{n}\bar{X}_2\right)$
$$= \bar{X}_1\left(1 - \frac{n_1}{n}\right) - \frac{n_2}{n}\bar{X}_2$$
$$= \frac{n_2}{n}\bar{X}_1 - \frac{n_2}{n}\bar{X}_2 = \frac{n_2}{n}(\bar{X}_1 - \bar{X}_2)$$

So that $(\bar{X}_1 - \bar{X}_.)^2 = \frac{n_2^2}{n^2}(\bar{X}_1 - \bar{X}_2)^2$, and
$(\bar{X}_2 - \bar{X}_.)^2 = \frac{n_1^2}{n^2}(\bar{X}_1 - \bar{X}_2)^2$, and

$$SSB = \frac{n_1 n_2^2}{n^2}(\bar{X}_1 - \bar{X}_2)^2 + \frac{n_2 n_1^2}{n^2}(\bar{X}_1 - \bar{X}_2)^2$$

$$= \frac{n_1 n_2}{n^2}(n_2+n_1)(\bar{X}_1 - \bar{X}_2)^2 = \frac{n_1 n_2}{n_1+n_2} \cdot \frac{\cancel{n_1+n_2}}{\cancel{n_1+n_2}}(\bar{X}_1 - \bar{X}_2)^2$$

Q1 continued

Have $F = \dfrac{SSB}{SSW/(n_1+n_2-2)} = \dfrac{SSB}{S_p^2}$

And $SSB = \left(\dfrac{n_1 n_2}{n_1+n_2}\right)(\bar{X}_1 - \bar{X}_2)^2$

Now $\dfrac{1}{n_1} + \dfrac{1}{n_2} = \dfrac{n_2+n_1}{n_1 n_2}$, so

$$F = \frac{SSB}{S_p^2} = \frac{(\bar{X}_1-\bar{X}_2)^2}{S_p^2 \, \frac{n_1+n_2}{n_1 n_2}} = \frac{(\bar{X}_1-\bar{X}_2)^2}{S_p^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}$$

$$= \left(\frac{\bar{X}_1-\bar{X}_2}{S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}\right)^2 = T^2$$

So $F = T^2$ not just in distribution, but in the actual formula.

(2) $\theta = (\mu_1, \mu_2, \sigma^2)$

$\hat{\Theta} = (\bar{X}, \bar{Y}, \hat{\sigma}^2)$, $\hat{\Theta}_0 = (\hat{\mu}_0, \hat{\mu}_0, \hat{\sigma}_0^2)$

$\hat{\mu}_0 = \frac{1}{n_1+n_2}\left(\sum_{i=1}^{n_1} X_i + \sum_{i=1}^{n_2} Y_i\right)$, The overall sample mean

$$\hat{\sigma}_0^2 = \frac{1}{n_1+n_2}\left(\sum_{i=1}^{n_1}(X_i - \hat{\mu}_0)^2 + \sum_{i=1}^{n_2}(Y_i - \hat{\mu}_0)^2\right)$$

The usual MLE on the whole sample

To get $\hat{\sigma}^2$

$$l(\sigma) = \ln\left[\prod_{i=1}^{n_1} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}(x_i-\bar{x})^2} \prod_{i=1}^{n} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}(y_i-\bar{y})^2}\right]$$

$$= \ln\left[\sigma^{-(n_1+n_2)} (2\pi)^{-\frac{n_1+n_2}{2}} \operatorname{Exp} -\frac{1}{2\sigma^2}\left(\sum_{i=1}^{n}(x_i-\bar{x})^2 + \sum_{i=1}^{n}(y_i-\bar{y})^2\right)\right]$$

$$= -(n_1+n_2)\ln\sigma - \frac{n_1+n_2}{2}\ln 2\pi - \frac{1}{2}\left(n_1\hat{\sigma}_1^2 + n_2\hat{\sigma}_2^2\right)\sigma^{-2}$$

$$l'(\sigma) = -\frac{n_1+n_2}{\sigma} - \frac{1}{2}\left(n_1\hat{\sigma}_1^2 + n_2\hat{\sigma}_2^2\right)(-2)\sigma^{-3} \overset{set}{=} 0$$

$$\Rightarrow \frac{n_1+n_2}{\cancel{\sigma}} = \frac{n_1\hat{\sigma}_1^2 + n_2\hat{\sigma}_2^2}{\cancel{\sigma}\,\sigma^2}$$

$$\Rightarrow \hat{\sigma}^2 = \frac{n_1\hat{\sigma}_1^2 + n_2\hat{\sigma}_2^2}{n_1+n_2} = \frac{SSW}{n_1+n_2}$$

2 cont.

$$\lambda(\underset{\sim}{x},\underset{\sim}{y}) = \frac{\prod_{i=1}^{n_1} \frac{1}{\hat{\sigma}_0\sqrt{2\pi}} e^{-\frac{1}{2\hat{\sigma}_0^2}(x_i-\hat{\mu}_0)^2} \prod_{i=1}^{n_2} \frac{1}{\hat{\sigma}_0\sqrt{2\pi}} e^{-\frac{1}{2\hat{\sigma}_0^2}(y_i-\hat{\mu}_0)^2}}{\prod_{i=1}^{n} \frac{1}{\hat{\sigma}\sqrt{2\pi}} e^{-\frac{1}{2\hat{\sigma}^2}(x_i-\bar{x})^2} \prod_{i=1}^{n_2} \frac{1}{\hat{\sigma}\sqrt{2\pi}} e^{-\frac{1}{2\hat{\sigma}^2}(y_i-\bar{y})^2}}$$

$$= \left(\frac{\hat{\sigma}^2}{\hat{\sigma}_0^2}\right)^{\frac{n_1+n_2}{2}} \frac{\mathrm{Exp} - \frac{1}{2\hat{\sigma}_0^2}\left(\sum_{i=1}^{n_1}(x_i-\hat{\mu}_0)^2 + \sum_{i=1}^{n_2}(y_i-\hat{\mu}_0)^2\right)}{\mathrm{Exp} - \frac{1}{2\hat{\sigma}^2}\left(n_1\hat{\sigma}_1^2 + n_2\hat{\sigma}_2^2\right)}$$

$$= \left(\frac{\hat{\sigma}^2}{\hat{\sigma}_0^2}\right)^{\frac{n_1+n_2}{2}} \frac{\mathrm{Exp} - \frac{1}{2\hat{\sigma}_0^2}(n_1+n_2)\hat{\sigma}_0^2}{\mathrm{Exp} - \frac{1}{2\hat{\sigma}^2}(n_1+n_2)\hat{\sigma}^2} = \left(\frac{\hat{\sigma}^2}{\hat{\sigma}_0^2}\right)^{\frac{n_1+n_2}{2}}$$

So $C_k = \left\{(\underset{\sim}{x},\underset{\sim}{y}) : \left(\frac{\hat{\sigma}^2}{\hat{\sigma}_0^2}\right)^{\frac{n_1+n_2}{2}} \le k\right\}$ where $0<k<1$

$$= \left\{(\underset{\sim}{x},\underset{\sim}{y}) : \frac{\hat{\sigma}^2}{\hat{\sigma}_0^2} \le k^{\frac{2}{n_1+n_2}} = k_1\right\}$$

$$= \left\{(\underset{\sim}{x},\underset{\sim}{y}) : \frac{\cancel{\frac{1}{n_1+n_2}}\left(\sum_{i=1}^{n_1}(x_i-\bar{x})^2 + \sum_{i=1}^{n}(y_i-\bar{y})^2\right)}{\cancel{\frac{1}{n_1+n_2}}\left(\sum_{i=1}^{n_1}(x_i-\hat{\mu}_0)^2 + \sum_{i=1}^{n_2}(y_i-\hat{\mu}_0)^2\right)} \le k_1\right\}$$

By a familiar calculation,

$$\sum_{i=1}^{n_1}(x_i-\hat{\mu}_0)^2 = \sum_{i=1}^{n_1}(x_i-\bar{x})^2 + n_1(\bar{x}-\hat{\mu}_0)^2$$

$$\sum_{i=1}^{n_2}(y_i-\hat{\mu}_0)^2 = \sum_{i=1}^{n_2}(y_i-\bar{y})^2 + n_2(\bar{y}-\hat{\mu}_0)^2$$

(2 cont) So, taking reciprocal of both sides,

$$C_k = \left\{ (\underset{\sim}{x}, \underset{\sim}{y}) : \frac{\sum_{i=1}^{n_1}(x_i-\bar{x})^2 + \sum_{i=1}^{n_2}(y_i-\bar{y})^2 + n_1(\bar{x}-\hat{\mu}_0)^2 + n_2(\bar{y}-\hat{\mu}_0)^2}{\sum_{i=1}^{n_1}(x_i-\bar{x})^2 + \sum_{i=1}^{n_2}(y_i-\bar{y})^2} \geq \frac{1}{k_1} = k_2 \right\}$$

$$= \left\{ (\underset{\sim}{x}, \underset{\sim}{y}) : 1 + \frac{n_1(\bar{x}-\hat{\mu}_0)^2 + n_2(\bar{y}-\hat{\mu}_0)^2}{\sum_{i=1}^{n_1}(x_i-\bar{x})^2 + \sum_{i=1}^{n_2}(y_i-\bar{y})^2} \geq k_2 \right\}$$

$$= \left\{ (x, y) : \frac{n_1(\bar{x}-\hat{\mu}_0)^2 + n_2(\bar{y}-\hat{\mu}_0)^2}{(n_1+n_2-2)S_p^2} \geq k_2 - 1 = k_3 \right\}$$

$$= \left\{ (\underset{\sim}{x}, \underset{\sim}{y}) : \frac{n_1(\bar{x}-\hat{\mu}_0)^2 + n_2(\bar{y}-\hat{\mu}_0)^2}{S_p^2} \geq (n_1+n_2-2)k_3 = k_4 \right\}$$

(Repeating the work from Q1 on P. 1, recognizing $\hat{\mu}_0 = \bar{X}_\cdot$,)

$$= \left\{ (\underset{\sim}{x}, \underset{\sim}{y}) : \frac{\frac{n_1 n_2}{n_1+n_2}(\bar{x}-\bar{y})^2}{S_p^2} \geq k_4 \right\}$$

$$= \left\{ (\underset{\sim}{x}, \underset{\sim}{y}) : \frac{(\bar{x}-\bar{y})^2}{S_p^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)} \geq k_4 \right\}$$

$$= \left\{ (\underset{\sim}{x}, \underset{\sim}{y}) : \left| \frac{\bar{x}-\bar{y}}{s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} \right| \geq \sqrt{k_4} = k_5 \right\}$$

$$= \left\{ (\underset{\sim}{x}, \underset{\sim}{y}) : |t| \geq k_5 \right\}$$

For a size α test, let $k_5 = t_{1-\alpha/2}(n_1+n_2-2)$

The two-sample t-test

③ a) Find MLE first

$\boxed{6}$

$$L(\theta) = \prod_{i=1}^{n} \frac{1}{2\theta^3} x_i^2 e^{-x_i/\theta} = \frac{1}{6^n}\theta^{-3n}\prod_{i=1}^{n} x_i^2 \, e^{-\frac{1}{\theta}\sum_{i=1}^{n} x_i}$$

$$\ell'(\theta) = \frac{d}{d\theta}\left(-n\ln 2 - 3n\ln\theta + \ln\prod_{i=1}^{n} x_i^2 - n\bar{x}\theta^{-1}\right)$$

$$= -\frac{3n}{\theta} - n\bar{x}(-1)\theta^{-2} \overset{\text{set}}{=} 0 \Rightarrow \frac{3n}{\theta} = \frac{n\bar{x}}{\theta^2}$$

$$\Rightarrow \theta = \frac{n\bar{x}}{3n} = \frac{\bar{x}}{3}$$

2nd derivative test (not on final exam)

$$\ell''(\theta) = \frac{d}{d\theta}\left(-3n\theta^{-1} + n\bar{x}\theta^{-2}\right) = -3n(-1)\theta^{-2} + n\bar{x}(-2)\theta^{-3}$$

$$= \frac{3n}{\theta^2} - \frac{2n\bar{x}}{\theta^3} \quad \text{Evaluate at } \theta = \bar{x}/3 \text{, get}$$

$$\frac{3n}{(\bar{x}/3)^2} - \frac{2n\bar{x}}{(\bar{x}/3)^3} = \frac{3n\cdot 9}{\bar{x}^2} - \frac{2n\cancel{\bar{x}}\cdot 27}{\bar{x}^2\cancel{\bar{x}}}$$

$$= \frac{27n}{\bar{x}^2} - \frac{54n}{\bar{x}^2} < 0 \quad \text{concave down} \ \cap \ \text{max}$$

Model density is Gamma $(\alpha = 3, \lambda = \frac{1}{\theta})$

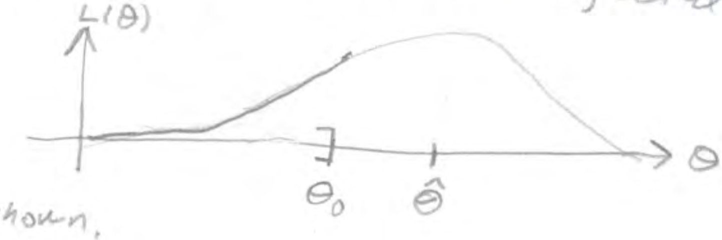
so $M_{X_i}(t) = (1-\theta t)^{-3}$, & $M_{\bar{X}}(t) = \left(1-\frac{\theta t}{n}\right)^{-3n}$

and $M_{\hat{\theta}}(t) = M_{\bar{X}/3}(t) = M_{\bar{X}}(t/3) = \left(1-\frac{\theta t}{3n}\right)^{-3n}$

MGF of Gamma $\left(\alpha = 3n, \lambda = \frac{3n}{\theta}\right)$

(3b) Need to find $\hat{\Theta}_0$. If $\hat{\Theta} = \frac{\bar{X}_n}{3} \le \theta_0$, $\hat{\Theta}_0 = \hat{\Theta}$, $\lambda = 1$, and H_0 is not rejected.

If $\hat{\Theta} > \theta_0$



$\hat{\Theta}_0 = \theta_0$ as shown.

$$\lambda(\underset{\sim}{x}) = \frac{\prod_{i=1}^{n} \frac{1}{2\theta_0^3} x_i^2 e^{-x_i/\theta_0}}{\prod_{i=1}^{n} \frac{1}{2\hat{\theta}^3} x_i^2 e^{-x_i/\hat{\theta}}} = \frac{\hat{\theta}^{3n}}{\theta_0^{3n}} \frac{1/2^n}{1/2^n} \frac{e^{-\frac{1}{\theta_0}\sum_{i=1}^{n} x_i}}{e^{-\frac{1}{\hat{\theta}} n\bar{x}}}$$

$$= \left(\frac{\hat{\theta}}{\theta_0}\right)^{3n} \frac{e^{-\frac{n\bar{x}}{\theta_0}}}{e^{-n\bar{x}/\bar{x}/3}} = \left(\frac{\hat{\theta}}{\theta_0}\right)^{3n} \frac{e^{-n\bar{x}/\theta_0}}{e^{-3n}}$$

$$= \frac{\bar{x}^{3n}}{(3\theta_0)^{3n}} \frac{e^{-n\bar{x}/\theta_0}}{e^{-3n}}, \text{ and}$$

$$C_k = \left\{ \underset{\sim}{x} \in \mathbb{R}^n : \frac{(\bar{x}^3 e^{-\bar{x}/\theta_0})^n}{\theta_0^{3n}\, 27^n\, e^{-3n}} \le k \right\}, \quad 0 < k < 1$$

$$= \left\{ \underset{\sim}{x} : \bar{x}^3 e^{-\bar{x}/\theta_0} \le (\theta_0^3 \cdot 27 \cdot e^{-3} k)^{1/n} = k_1 \right\}$$

want to isolate $\bar{x}$. Study the function

$$g(x) = x^3 e^{-x/\theta_0}$$

(3b continued) $g(x) = x^3 e^{-x/\theta_0}$ $\boxed{8}$

$$g'(x) = u'v + v'u = 3x^2 e^{-x/\theta_0} + x^3 e^{-x/\theta_0}\left(\frac{-1}{\theta_0}\right)$$

$$= x^2 e^{-x/\theta_0}\left(3 - \frac{x}{\theta_0}\right) = 0 \text{ at } x = 3\theta_0$$

and if $x > 3\theta_0 \Rightarrow \frac{x}{\theta_0} - 3 < 0$ and g is $\downarrow$

A decreasing function has a unique inverse that is decreasing.

$$C_* = \left\{ \underset{\sim}{x} : \bar{x}^3 e^{-\bar{x}/\theta_0} \le k_1 \right\} = \left\{ \underset{\sim}{x} : g(\bar{x}) \le k_1 \right\}$$

$$= \left\{ \underset{\sim}{x} : g^{-1}(g(\bar{x})) \ge g^{-1}(k_1) = k_2 \right\}$$

$$= \left\{ \underset{\sim}{x} : \bar{x} \ge k_2 \right\}$$

To make this a chi-squared test, need to transform $\bar{X}_n$. From part (a),

$M_{\bar{X}_n}(t) = \left(1 - \frac{\theta t}{n}\right)^{-3n}$. If $\theta = \theta_0$,

$Y = \frac{2n\bar{X}_n}{\theta_0}$ has mgf $(1-2t)^{-3n}$

$= (1-2t)^{-\frac{6n}{2}}$, MGF of $\chi^2(6n)$

and letting $k_2 = \chi^2_{1-\alpha}(6n)$ would give

$$P_{\theta_0}\left(Y > \chi^2_{1-\alpha}(6n)\right) = \alpha$$

To make this test sig α, show $P_\theta(Y > \chi^2_{1-\alpha}(6n))$ is an increasing function of θ.

(3 b continued)

$c = \chi^2_{1-\alpha}(6n)$

For $\theta \le \theta_0$, $P_\theta(Y > c)$

$$= P_\theta\left(\frac{2n\bar{X}_n}{\theta_0} > c\right) = P\left(\frac{2n\bar{X}_n}{\theta} > \frac{\theta_0 c}{\theta}\right)$$

$$= P\left(W > \frac{\theta_0 c}{\theta}\right), \text{ where } W \sim \chi^2(6n)$$

$$= 1 - F_W(\theta_0 c \theta^{-1}), \text{ and}$$

$$\frac{d}{d\theta}\left(1 - F_W(\theta_0 c \theta^{-1})\right)$$

$$= -f_W\left(\frac{c\theta_0}{\theta}\right) \cdot c\theta_0 (-1)\theta^{-2} > 0 \quad \text{increasing}$$

So if H_0 is rejected when $Y = \frac{2n\bar{X}_n}{\theta_0} > \chi^2_{1-\alpha}(6n)$,

$$\max_{\theta \le \theta_0} P_\theta(Y > \chi^2_{1-\alpha}(6n))$$

$$= P_{\theta_0}(Y > \chi^2_{1-\alpha}(6n)) = \alpha$$

And the test has significance level α.

4 (a) Need unrestricted $\hat{\theta}$

$$\ell(\theta) = \ln \prod_{i=1}^{n} \theta (1-\theta)^{x_i} = \ln\left[\theta^n (1-\theta)^{n\bar{x}}\right]$$

$$= n \ln\theta + n\bar{x} \ln(1-\theta)$$

$$\ell'(\theta) = \frac{n}{\theta} + \frac{n\bar{x}}{1-\theta}(-1) = \frac{n}{\theta} - \frac{n\bar{x}}{1-\theta} \overset{set}{=} 0$$

$$\Longrightarrow \frac{n}{\theta} = \frac{n\bar{x}}{1-\theta} \Longrightarrow 1-\theta = \bar{x}\theta \Longrightarrow 1 = \theta + \bar{x}\theta$$

$$\Longrightarrow 1 = \theta(1+\bar{x}) \Longrightarrow \hat{\theta} = \frac{1}{1+\bar{x}}$$ Don't bother with 2nd derivative test

$$\lambda(\underset{\sim}{x}) = \frac{\left(\frac{1}{2}\right)^n \left(1-\frac{1}{2}\right)^{n\bar{x}}}{\left(\frac{1}{1+\bar{x}}\right)^n \left(1-\frac{1}{1+\bar{x}}\right)^{n\bar{x}}} = \frac{\left(\frac{1}{2}\right)^{n(1+\bar{x})}}{\frac{1}{(1+\bar{x})^n}\left(\frac{1+\bar{x}-1}{1+\bar{x}}\right)^{n\bar{x}}}$$

$$= \frac{(1+\bar{x})^{n(1+\bar{x})}}{2^{n(1+\bar{x})}\,\bar{x}^{n\bar{x}}}$$, and

$$G^2 = -2\ln\left(\frac{(1+\bar{X}_n)^{1+\bar{X}_n}}{2^{(1+\bar{X}_n)}\,\bar{X}_n^{\bar{X}_n}}\right)^n$$

$$= -2n\left[(1+\bar{X}_n)\ln(1+\bar{X}_n) - (1+\bar{X}_n)\ln 2 - \bar{X}_n \ln \bar{X}_n\right]$$

$$= 2n\left[\bar{X}_n \ln \bar{X}_n - (1+\bar{X}_n)\left(\ln(1+\bar{X}_n) - \ln 2\right)\right]$$

(4b) $G_n^2 = 100\left(1.56 \ln(1.56) - 2.56(\ln 2.56 - \ln 2)\right)$

$= 6.17$

Critical value is $\chi^2_{0.95}(1) = 3.84$

c) Yes

d) $\hat{\theta} = \frac{1}{1+\bar{x}} = \frac{1}{1+1.56} = \frac{1}{2.56} = 0.39 < \frac{1}{2}$

Conclude $\theta < \frac{1}{2}$

$\boxed{12}$

(5) a)

$$G^2 = -2\ln \frac{\prod_{i=1}^{n} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x_i^2}}{\prod_{i=1}^{n} \frac{1}{\hat{\sigma}\sqrt{2\pi}} e^{-\frac{1}{2\hat{\sigma}^2}(x_i-\bar{x})^2}}$$

$$= -2\ln\left(\frac{e^{-\frac{1}{2}\sum_{i=1}^{n} x_i^2}}{\frac{1}{(\hat{\sigma}^2)^{n/2}} e^{-\frac{1}{2\hat{\sigma}^2} n\hat{\sigma}^2}} \right)$$

$$= -2\left[-\frac{1}{2}\sum_{i=1}^{n} x_i^2 - \ln\left((\hat{\sigma}^2)^{-n/2} e^{-\frac{n}{2}} \right) \right]$$

$$= -2\left[-\frac{1}{2}\sum_{i=1}^{n} x_i^2 + \frac{n}{2}\ln\hat{\sigma}^2 - -\frac{n}{2} \right]$$

$$= \sum_{i=1}^{n} x_i^2 - n\ln\hat{\sigma}^2 - n$$

Now $\hat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n}(X_i - \bar{X}_n)^2 = \frac{1}{n}\left(\sum X_i^2 - n\bar{X}_n^2\right)$

$$\Rightarrow n\hat{\sigma}^2 = \sum_{i=1}^{n} X_i^2 - n\bar{X}^2 \Rightarrow \sum_{i=1}^{n} X_i^2 = n(\hat{\sigma}^2 + \bar{X}_n^2)$$

and so

$$G^2 = n(\hat{\sigma}^2 + \bar{X}_n^2 - \ln\hat{\sigma}^2 - 1)$$

b i) $G^2 = 200(1.353 + 0.062^2 - \ln(1.353) - 1)$

$= 10.9$

ii) $\chi^2_{0.95}(2) = 5.99$

iii) Yes

iv) No

(6) (a) $\Omega = \{(\lambda_1, \dots, \lambda_6) : \lambda_j > 0\}$

(b) $H_0: \lambda_1 = \dots = \lambda_6$

(c) $H_1: \lambda_j \neq \lambda_k$, some $j \neq k$, $j = 1, \dots 6$, $k = 1, \dots, 6$

(d) $\Omega_0 = \{(\lambda_1, \dots, \lambda_6) : 0 < \lambda_1 = \lambda_2 = \dots = \lambda_6\}$

(e) $\Omega_1 = \Omega \cap \Omega_0^c$ A cheap but correct answer.

(f) $\hat{\theta} = (10.68, 9.87234, 9.56, 8.52, 10.48571, 9.98)$

(g) If all λ parameters are equal, it's one big Poisson random sample, and $\hat{\lambda} = \bar{y}_{\cdot}$

$$\hat{\lambda}_0 = \left[(10.68)50 + \cdots (9.88)50\right]/282 = 9.82 = \bar{y}_{\cdot}$$

$$\hat{\theta}_0 = (9.82, 9.82, 9.82, 9.82, 9.82, 9.82)$$

(h)
$$G_n^2 = -2 \ln \frac{\prod_{j=1}^{k} \prod_{i=1}^{n_j} \frac{e^{-\bar{y}_{\cdot}} \bar{y}_{\cdot}^{y_{ij}}}{y_{ij}!}}{\prod_{j=1}^{k} \prod_{i=1}^{n_j} \frac{e^{-\bar{y}_j} \bar{y}_j^{y_{ij}}}{y_{ij}!}}$$

$$= -2 \ln \frac{e^{-n\bar{y}_{\cdot}} \bar{y}_{\cdot}^{\sum_{j=1}^{k}\sum_{i=1}^{n_j} y_{ij}}}{\prod_{j=1}^{k} e^{n_j \bar{y}_j} \bar{y}_j^{\sum_{i=1}^{n_j} y_{ij}}}$$

$$= -2\ln \frac{e^{-n\bar{y}_{\circ}}\ \bar{y}_{\circ}^{\,n\bar{y}_{\circ}}}{e^{\sum_{j=1}^{k} n_j \bar{y}_j} \prod_{j=1}^{k} \bar{y}_j^{\,n_j \bar{y}_j}}$$

$$= -2\ln \frac{\cancel{e^{-n\bar{y}_{\circ}}}\ \bar{y}_{\circ}^{\,n\bar{y}_{\circ}}}{\cancel{e^{-n\bar{y}_{\circ}}} \prod_{j=1}^{k} \bar{y}_j^{\,n_j \bar{y}_j}}$$

$$= -2\left(n\bar{y}_{\circ} \ln \bar{y}_{\circ} - \sum_{j=1}^{k} n_j \bar{y}_j \ln \bar{y}_j\right)$$

$$= 2\left(\sum_{j=1}^{k} n_j \bar{y}_j \ln \bar{y}_j - n\bar{y}_{\circ} \ln \bar{y}_{\circ}\right)$$

(i) $n\bar{y}_{\circ} \ln \bar{y}_{\circ} = 282 * 9.8156 * \ln 9.8156 = 6322.035$

$\sum_{j=1}^{k} n_j \bar{y}_j \ln \bar{y}_j = 50 * 10.68 \ln 10.68 + 47 * 9.87234 * \ln 9.87234 + \cdots + 50 * 9.98 * \ln 9.98$

$= 6329.391$, so

$G_n^2 = 2(6329.391 - 6322.035) = 14.71$

(j) $df = 5$

(k) $\chi^2_{0.95}(5) = 11.07$

(l) Yes

(m) No