STA 260s20 Assignment One: Mostly Review

These homework problems are not to be handed in. They are preparation for Quiz 1 and Term Test 1. **Please try each question before looking at the solution**.

1. Let the continuous random variable X have density $f_X(x) = 2x\, e^{-x^2}\, I(x > 0)$.
   (a) Write the cumulative distribution function $F_X(x)$ using indicator functions. Show your work.
   (b) Calculate $P(X > \frac{1}{2})$. My answer is 0.7788.

2. The discrete random variable X has probability mass function

$$p_X(x) = \frac{|x|}{20} I(x = -4, \ldots, 4).$$

   Let $Y = X^2 - 1$.
   (a) What is $E(X)$? The answer is a number. Show some work. My answer is zero.
   (b) Calculate the variance of X. The answer is a number. My answer is 10.
   (c) What is $P(Y = 8)$? My answer is 0.30
   (d) What is $P(Y = -1)$? My answer is zero.
   (e) What is $P(Y = -4)$? My answer is zero.
   (f) What is the probability distribution of Y? Give the y values with their probabilities for y with $p_Y(y) > 0$.

y	0	3	8	15
p(y)	0.1	0.2	0.3	0.4

   (g) What is $E(Y)$? The answer is a number. My answer is 9.
   (h) What is $Var(Y)$? The answer is a number. My answer is 30.

3. Let $f_X(x) = \frac{1}{2} I(-1 < x < 1)$, and $Y = X^2$. Find $f_Y(y)$. This is a valuable workout in the use of indicator functions.

4. Let $X \sim N(\mu, \sigma^2)$. Show $Z = \frac{X-\mu}{\sigma} \sim N(0, 1)$.

5. Let $X_1, \ldots X_n$ be independent and identically distributed $N(\mu, \sigma^2)$ random variables. Find the distribution of $Y = a + \sum_{i=1}^n b_i X_i$. Show your work.

6. Let $Z \sim N(0, 1)$. Show $Z^2 \sim \chi^2(1)$.

7. Let $Y_1, \ldots Y_n$ be independent $\chi^2(\nu_i)$ random variables. Show $Y = \sum_{i=1}^n Y_i \sim \chi^2 \left(\sum_{i=1}^n \nu_i\right)$.

8. Let $X_1, \ldots X_n$ be independent random variables with expected value μ and variance σ^2, and denote the sample mean by $\overline{X}_n = \frac{1}{n} \sum_{i=1}^n X_i$.
   (a) Calculate $E(\overline{X}_n)$. Show your work.
   (b) Calculate $Var(\overline{X}_n)$. Show your work.

9. The discrete random variables X and Y have joint distribution

	$x=1$	$x=2$	$x=3$
$y=1$	$3/12$	$1/12$	$3/12$
$y=2$	$1/12$	$3/12$	$1/12$

   (a) Calculate $Cov(X,Y)$. Show your work.

   (b) Are X and Y independent? Answer Yes or No and prove it.

10. Starting with the definition, show $Var(X) = E(X^2) - [E(X)]^2$.

11. Starting with the definition, show $Cov(X,Y) = E(XY) - E(X)E(Y)$.

12. Let X and Y be discrete random variables. Either prove that the following proposition is true in general, or show that it is not by giving a simple counter-example: If X and Y are independent, then $Cov(X,Y) = 0$.

13. Let X and Y be discrete random variables. Either prove that the following proposition is true in general, or show that it is not by giving a simple counter-example: If $Cov(X,Y) = 0$, then X and Y are independent.

14. Find $Cov(X, Y+Z)$. Use the definition of covariance. What fact on the formula sheet could you have used instead?

15. Let the random variable X have distribution function $F_{_X}(x) = 1$ for all real x. Is this possible? Answer Yes or No and briefly explain.

16. Let the continuous random variable X have density $f_{_X}(x)$. What's wrong with this?

$$F_{_X}(x) = \int_{-\infty}^{\infty} f_{_X}(t)\, dt$$

17. What's wrong with this? $F_{_{X|Y}}(x|y) = \frac{F_{_{X,Y}}(x,y)}{F_{_Y}(y)}$. To see it more easily, let X and Y be discrete.

18. Let X be a continuous random variable. Either prove that the following proposition is true in general, or show that it is not by giving a simple counter-example: $E\left(\frac{1}{X}\right) = \frac{1}{E(X)}$.

19. What's wrong with this? $Var(X) = E\left((X-\mu)^2\right) = \left(E(X-\mu)\right)^2 = \left(E(X) - E(\mu)\right)^2 = (\mu - \mu)^2 = 0$.
