

# Sets<sup>1</sup>

STA 256: Fall 2019

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# Statistical Experiment

This vocabulary is not in the text

A statistical experiment is a procedure whose outcome is not known in advance with certainty.

Sample Space: set of outcomes  $s \in S$

- Sell 500 lottery tickets, pick the winning number.

$$S = \{1, 2, \dots, 500\}$$

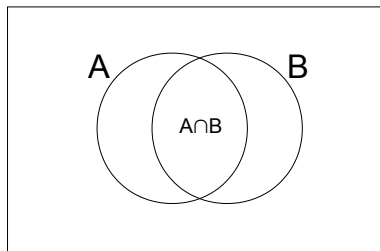
- Hold your breath as long as you can.

$$S = \{t : t \geq 0\}$$

- Pick coin or die from jar, roll or toss.

$$S = \{H, T, 1, 2, 3, 4, 5, 6\}$$

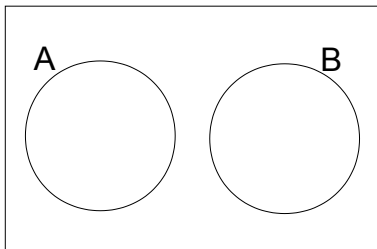
Event: Set of outcomes,  $A \subset S$



- $A \cap B = \{s \in S : s \in A \text{ and } s \in B\}$
- $A \cup B = \{s \in S : s \in A \text{ or } s \in B\}$
- $A^c = \{s \in S : s \notin A\}$
- $A \cap B^c = \{s \in S : s \in A \text{ and } s \notin B\}$

# Disjoint sets

- $A$  and  $B$  are said to be *disjoint* if  $A \cap B = \emptyset$
- The idea is that  $A$  and  $B$  have no elements in common; they do not overlap.



- However, recall that the null set is a subset of every set:  $\emptyset \subseteq A$ .
- So  $\emptyset \cap A = \emptyset$ .
- And the null set is also disjoint from every set.

# Set Laws

No proofs, just Venn diagrams at most

- Commutative:  $A \cup B = B \cup A$ ,  $A \cap B = B \cap A$
- Associative
  - $(A \cup B) \cup C = A \cup (B \cup C)$ ,
  - $(A \cap B) \cap C = A \cap (B \cap C)$
- Distributive (like multiplication)
  - $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
  - $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

# De Morgan Laws

Not in the text

- $(A \cap B)^c = A^c \cup B^c$
- $(A \cup B)^c = A^c \cap B^c$
- Rule: complement and flip  $\cup \cap$

# Extend the notation to larger number of sets

Not in the text

Distributive laws

- $A \cap \left( \bigcup_{j=1}^n B_j \right) = \bigcup_{j=1}^n (A \cap B_j)$ , or even

- $A \cap \left( \bigcup_{j=1}^{\infty} B_j \right) = \bigcup_{j=1}^{\infty} (A \cap B_j)$

and

- $A \cup \left( \bigcap_{j=1}^n B_j \right) = \bigcap_{j=1}^n (A \cup B_j)$

- $A \cup \left( \bigcap_{j=1}^{\infty} B_j \right) = \bigcap_{j=1}^{\infty} (A \cup B_j)$

De Morgan Laws (complement and flip)

- $(\bigcap_{j=1}^{\infty} A_j)^c = \bigcup_{j=1}^{\infty} A_j^c$

- $(\bigcup_{j=1}^{\infty} A_j)^c = \bigcap_{j=1}^{\infty} A_j^c$

## WARNING!

- Addition and subtraction apply only to numbers. They are not set operations.
- For example, if  $A$  and  $B$  are sets, then  $A + B$  is not defined.
- $A + B$  is not the same as  $A \cup B$ .
- Because what is  $A + A - A$ ?
- Some very bad probability proofs use addition and subtraction of sets.
- Such proofs will receive a **zero**.



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<http://www.utstat.toronto.edu/~brunner/oldclass/256f19>