$$Z_n = \frac{\sqrt{n}(\bar{X}-\mu)}{\sigma} \qquad \mu = \frac{1}{2} \quad \sigma^2 = \frac{1}{12}$$

Sample Questions: Limits

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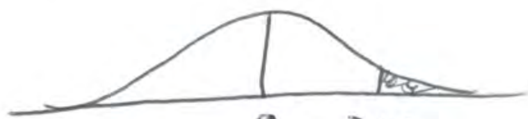
1. Let S be the sum of ~~12~~ 16 independent Uniform(0,1) random variables. Find the approximate $P(S > 12)$.

$$= P\left(\frac{\sum_{i=1}^{16} X_i}{n} > \frac{12}{16}\right) = P\left(\bar{X}_n > \frac{3}{4}\right)$$

$$= P\left\{\frac{\sqrt{n}(\bar{X}_n - \mu)}{\sigma} > \frac{\sqrt{16}\left(\frac{3}{4}-\frac{1}{2}\right)}{\sqrt{1/12}}\right\}$$

work

$$= P\{Z_n > 3.46\}$$



$$\approx 1 - \Phi(3.46) = 1 - 0.9997 = 0.0003$$

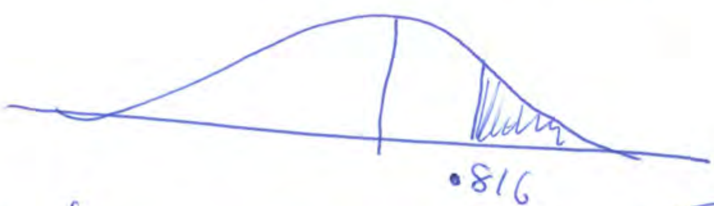
Phi CDF of $N(0,1)$

2. A multiple choice test has 50 questions with answers ABCD. If a student answers completely at random, what are the chances of getting 30% or better? You may use the fact that a Bernoulli(p) has expected value p and variance $p(1-p)$.

$$Z_n = \frac{\sqrt{n}\,(\bar{X}_n - \mu)}{\sigma}$$

$$P(\bar{X}_n \geq .3) = P\left\{ \frac{\sqrt{n}(\bar{X}-\mu)}{\sigma} \geq \frac{\sqrt{50}(.3 - .25)}{\sqrt{.25(1-.25)}} \right\}$$

$$= P(Z_n \geq 0.816) \approx 1 - \Phi(0.816)$$



$$= 1 - .7939 = .2061$$

a) What is the probability that the average time for 25 patients is more than 6 minutes?

(b)

3. In a walk-in medical clinic, the time a doctor spends per patient (including paperwork) comes from an unfamiliar skewed distribution with mean 5.1 and standard deviation 4.8 minutes. Find the maximum number of patients that should be scheduled so that the probability of working more than a 7 hour day will be less than 5%.

$\mu = 5.1, \ \sigma = 4.8$

$$a)\ P(\bar{X}_n > 6) = P\left\{ \frac{\sqrt{n}(\bar{X}_n - \mu)}{\sigma} > \frac{\sqrt{25}(6 - 5.1)}{4.8} \right\}$$

$$= P(Z_n > 0.94) \approx 1 - \Phi(0.94)$$

$$= 1 - .8264 = 0.1736$$

$$\sum_{i=1}^{n} X_i$$

$$.05 \overset{\text{want}}{=} P(S > 420)$$

$\mu = 5.1, \quad \sigma = 4.8 \quad S = \sum_{i=1}^{n} X_i$

$$.05 = P\left(\sum_{i=1}^{n} X_i > 420\right) = P(\bar{X}_n > 420/n)$$

$$= P\left\{\frac{\sqrt{n}(\bar{X}_n - \mu)}{\sigma} > \frac{\sqrt{n}(420/n - 5.1)}{4.8}\right\}$$

$$= P\left\{Z_n > \frac{\sqrt{n}(420/n - 5.1)}{4.8}\right\}$$

$$\approx 1 - \Phi\left(\frac{\sqrt{n}(420/n - 5.1)}{4.8}\right) = .05$$

$$\Longleftrightarrow \Phi\left(\frac{\sqrt{n}(420/n - 5.1)}{4.8}\right) = .95$$

$\Longleftrightarrow$

$$\frac{\sqrt{n}(420/n - 5.1)}{4.8} = \Phi^{-1}(.95) = 1.645$$

$$\Longleftrightarrow \sqrt{n}(420/n - 5.1) = (4.8)(1.645)$$

$$\Longleftrightarrow \frac{420}{\sqrt{n}} - 5.1\sqrt{n} = (4.8)(1.645)$$

$$\Longleftrightarrow \quad 420 - 5.1n = (4.8)(1.645)\sqrt{n}$$

$$\Longleftrightarrow \quad \underbrace{5.1}_{A}\, n + \underbrace{(4.8)(1.645)}_{B}\sqrt{n}\ \underbrace{-420}_{C} = 0$$

$$\frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \text{ get } \sqrt{n} = 8.333692$$

$$\text{or } -9.881927$$

So $n = 8.33\text{etc}^2 = 69.45$

Let $n = 69$