

# Independence<sup>1</sup>

STA 256: Fall 2018

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## Independence: The idea

- Independent means totally unrelated.
- If we say that taking Vitamin C supplements is *independent* of whether you get cancer, it means that taking Vitamin C supplements has *no connection* to whether you get cancer or not.
- It's a strong statement.
- It has a precise technical definition.

## Definition of independence

Suppose  $P(B) > 0$ , and  $P(A|B) = P(A)$ .

$$\begin{aligned}\frac{P(A \cap B)}{P(B)} &= P(A) \\ \Rightarrow P(A \cap B) &= P(A) P(B)\end{aligned}$$

We use this *definition*. We say the events  $A$  and  $B$  are independent when

$$P(A \cap B) = P(A) P(B)$$

It's symmetric, and applies even if  $P(A) = 0$  or  $P(B) = 0$ .

## Definition of *mutual* independence

### Of several events

A set of events  $A_1, \dots, A_n$  are *mutually independent* if the probability of the intersection of any sub-collection is the product of probabilities.

Pairwise is not enough. Example from the text, P. 24:

A fair coin is tossed twice. Outcomes are HH, HT, TH, TT

Let

$A$  = Head on first toss.

$B$  = Head on second toss.

$C$  = Exactly one Head.

$P(A) = P(B) = P(C) = \frac{1}{2}$  and they are pairwise independent, but  $P(A \cap B \cap C) = P(\emptyset) = 0 \neq \frac{1}{8}$ .

## The dice have no memory

Successive outcomes of simple statistical experiments like flipping a coin or rolling a die will always be assumed mutually independent.

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<http://www.utstat.toronto.edu/~brunner/oldclass/256f18>