Independence[1]

STA 256: Fall 2018

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Independence: The idea

- Independent means totally unrelated.
- If we say that taking Vitamin C supplements in *independent* of whether you get cancer, it means that taking Vitamin C supplements has *no connection* to whether you get cancer or not.
- It's a strong statement.
- It has a precise technical definition.

Definition of independence

Suppose $P(B) > 0$, and $P(A|B) = P(A)$.

$$\frac{P(A \cap B)}{P(B)} = P(A)$$
$$\Rightarrow \quad P(A \cap B) = P(A)\,P(B)$$

We use this *definition*. We say the events A and B are independent when

$$P(A \cap B) = P(A)\,P(B)$$

It's symmetric, and applies even if $P(A) = 0$ or $P(B) = 0$.

Definition of *mutual* independence
Of several events

A set of events $A_1, \ldots, A_n$ are *mutually independent* if the probability of the intersection of any sub-collection is the product of probabilities.

Pairwise is not enough. Example from the text, P. 24:

A fair coin is tossed twice. Outcomes are HH, HT, TH, TT
Let

A = Head on first toss.

B = Head on second toss.

C = Exactly one Head.

$P(A) = P(B) = P(C) = \frac{1}{2}$ and they are pairwise independent, but $P(A \cap B \cap C) = P(\emptyset) = 0 \neq \frac{1}{8}$.

The dice have no memory

Successive outcomes of simple statistical experiments like flipping a coin or rolling a die will always be assumed mutually independent.

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