

## Sample Questions: Expected Value, Variance and Covariance

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1. Let  $X$  have a continuous uniform distribution on  $(a, b)$ . Calculate  $E(X)$ .

2. Let  $X \sim \text{Poisson}(\lambda)$ . Calculate  $E(X)$ .

3. Let the continuous random variable  $X$  have density  $f(x) = \begin{cases} \frac{1}{x^2} & \text{for } x \geq 1 \\ 0 & \text{otherwise} \end{cases}$

(a) Verify that  $f(x)$  integrates to one.

(b) Calculate  $E(X)$ .

4. Let  $X \sim N(\mu, \sigma)$ . Calculate  $E(X)$ .

5. Let  $X$  have a binomial distribution with parameters  $n$  and  $p$ . Calculate  $E(X)$ .

- Let  $X$  have a Gamma distribution with parameters  $\alpha$  and  $\lambda$ . Calculate  $E(X^k)$ .

7. Let  $X$  and  $Y$  be independent (continuous) random variables. Show  $E(XY) = E(X)E(Y)$ .

8. Prove  $\text{Var}(a + X) = \text{Var}(X)$ .



9. Prove  $\text{Var}(bX) = b^2\text{Var}(X)$ .

10. Show  $Var(X) = E(X^2) - [E(X)]^2$ .

11. Let  $X \sim \text{Uniform}(0,1)$ . Calculate  $\text{Var}(X)$ .

12. Let  $X$  have density  $e^{-x}$  for  $x \geq 0$  and zero otherwise. Calculate  $Var(X)$ .

13. Let  $X \sim N(\mu, \sigma)$ . Calculate  $Var(X)$ .

14. The discrete random variables  $x$  and  $y$  have joint distribution

	$x = 1$	$x = 2$	$x = 3$
$y = 1$	$3/12$	$1/12$	$3/12$
$y = 2$	$1/12$	$3/12$	$1/12$

(a) What is  $E(X|Y = 1)$ ?

(b) What is  $E(Y^2|X = 2)$ ?

	$x = 1$	$x = 2$	$x = 3$
$y = 1$	$3/12$	$1/12$	$3/12$
$y = 2$	$1/12$	$3/12$	$1/12$

15. Let  $f_{x,y}(x, y) = 3$  for  $0 < x < 1$  and  $0 < y < x^2$ , and zero otherwise.

(a) Using  $f_x(x) = 3x^2$  for  $0 < x < 1$ , what is  $f_{y|x}(y|x)$ ? Don't forget the support.

(b) Find  $E(Y|X = x)$ , where  $x$  is a fixed constant between zero and one.

(c) Find  $E(Y)$  by double expectation.

(d) Using  $f_y(y) = 3(1 - y^{1/2})$  for  $0 < x < 1$ , calculate  $E(Y)$  directly.



16. Show that  $Cov(X, Y) = E(XY) - E(X)E(Y)$

17. Show that if  $X$  and  $Y$  are independent,  $Cov(X, Y) = 0$ .

18. The discrete random variables  $x$  and  $y$  have joint distribution

	$x = 1$	$x = 2$	$x = 3$
$y = 1$	$3/12$	$1/12$	$3/12$
$y = 2$	$1/12$	$3/12$	$1/12$

(a) Find  $Cov(X, Y)$ .

(b) Are  $X$  and  $Y$  independent?

19. Show  $Var(aX + bY) = a^2Var(X) + b^2Var(Y) + 2abCov(X, Y)$

20. Prove that  $Var(\sum_{i=1}^n a_i X_i) = \sum_{i=1}^n a_i^2 Var(X_i) + 2 \sum_{i=1}^{n-1} \sum_{j=i+1}^n a_i b_j Cov(X_i, X_j)$ .

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<http://www.utstat.toronto.edu/~brunner/oldclass/256f18>