Sample Questions: Expected Value, Variance and Covariance

STA256 Fall 2018.

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1. Let X have a continuous uniform distribution on (a, b). Calculate $E(X)$.

2. Let $X \sim \text{Poisson}(\lambda)$. Calculate $E(X)$.

3. Let the continuous random variable X have density $f(x) = \begin{cases} \frac{1}{x^2} & \text{for } x \geq 1 \\ 0 & \text{otherwise} \end{cases}$

(a) Verify that $f(x)$ integrates to one.

(b) Calculate $E(X)$.

4. Let $X \sim N(\mu, \sigma)$. Calculate $E(X)$.

5. Let X have a binomial distribution with parameters n and p. Calculate $E(X)$.

6. Let X have a Gamma distribution with parameters α and λ. Calculate $E(X^k)$.

7. Let X and Y be independent (continuous) random variables. Show $E(XY) = E(X)E(Y)$.

8. Prove $Var(a + X) = Var(X)$.

9. Prove $Var(bX) = b^2Var(X)$.

10. Show $Var(X) = E(X^2) - [E(X)]^2$.

11. Let $X \sim$ Uniform(0,1). Calculate $Var(X)$.

12. Let X have density e^{-x} for $x \geq 0$ and zero otherwise. Calculate $Var(X)$.

13. Let $X \sim N(\mu, \sigma)$. Calculate $Var(X)$.

14. The discrete random variables x and y have joint distribution

	$x = 1$	$x = 2$	$x = 3$
$y = 1$	$3/12$	$1/12$	$3/12$
$y = 2$	$1/12$	$3/12$	$1/12$

(a) What is $E(X|Y = 1)$?

(b) What is $E(Y^2|X = 2)$?

	$x = 1$	$x = 2$	$x = 3$
$y = 1$	$3/12$	$1/12$	$3/12$
$y = 2$	$1/12$	$3/12$	$1/12$

15. Let $f_{x,y}(x, y) = 3$ for $0 < x < 1$ and $0 < y < x^2$, and zero otherwise.

(a) Using $f_x(x) = 3x^2$ for $0 < x < 1$, what is $f_{y|x}(y|x)$? Don't forget the support.

(b) Find $E(Y|X = x)$, where x is a fixed constant between zero and one.

(c) Find $E(Y)$ by double expectation.

(d) Using $f_y(y) = 3(1 - y^{1/2})$ for $0 < x < 1$, calculate $E(Y)$ directly.

16. Show that $Cov(X, Y) = E(XY) - E(X)E(Y)$

17. Show that if X and Y are independent, $Cov(X, Y) = 0$.

18. The discrete random variables x and y have joint distribution

	$x = 1$	$x = 2$	$x = 3$
$y = 1$	$3/12$	$1/12$	$3/12$
$y = 2$	$1/12$	$3/12$	$1/12$

(a) Find $Cov(X, Y)$.

(b) Are X and Y independent?

19. Show $Var(aX + bY) = a^2Var(X) + b^2Var(Y) + 2abCov(X, Y)$

20. Prove that $Var\left(\sum_{i=1}^n a_iX_i\right) = \sum_{i=1}^n a_i^2 Var(X_i) + 2\sum_{i=1}^{n-1}\sum_{j=i+1}^n a_ib_jCov(X_i,X_j)$.
