Sample Questions: Counting Methods for Computing Probabilities

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1. Using the formula for $\binom{n}{r}$ from the formula sheet, and the Multiplication Principle, prove that the number of ways that n objects can be divided into r subsets with n_i objects in set i is $\binom{n}{n_1 \cdots n_r} = \frac{n!}{n_1! \cdots n_r!}$.

2. Sample r balls from a jar containing n numbered balls. How many outcomes are there is the sampling is

   (a) With replacement?

   (b) Without replacement?

3. Using the formula for ${}_nP_r$ from the formula sheet, and the Multiplication Principle, prove $\binom{n}{r} = \frac{n!}{r!\,(n-r)!}$.

4. A jar contains 10 red balls and 20 blue balls. If 5 balls are randomly sampled without replacement, what is the probability of

   (a) All blue?

   (b) Two red and three blue?

   (c) At least one red?

(d) A jar contains 10 red balls and 20 blue balls. If 5 balls are randomly sampled without replacement, what is the probability of obtaining k red balls, $k = 0, \ldots, 5$?

5. A shipment of n electronic components has k defectives. If we sample m components without replacement, what is the probability of observing at least one defective?

6. In how many ways can 20 basketball players be divided into 4 teams of 5?

7. In how many ways can 6 red flags, 2 blue flags and 4 yellow flags be arranged? The flags are indistinguishable.

8. A standard deck of 52 cards has four "suits:" spades, diamonds, hearts and clubs. Within each suit, the face values of the 13 cards are 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, King, Ace. A "hand" of poker is 5 cards, selected randomly without replacement.

   (a) A "flush" is a hand with 5 cards all of the same suit. What is the probability of a flush?

   (b) A "straight" is a hand in which the 5 cards are in sequence. Suit is ignored. An Ace can be either high or low. What is the probability of a straight?
