STA 256f18 Assignment Seven[1]

Please look at Sections 3.4 through 3.7 in Chapter 3 of the textbook and look over your lecture notes. In the text, don't worry about Bayesian inference and note that in Section 3.7, we are only covering the density of the maximum and minimim, not order statistics in general.

These homework problems are not to be handed in. They are preparation for Term Test 3 and the final exam. All textbook problems are from Chapter Three. Use the formula sheet to do the problems. On tests and the final exam, you may use anything on the formula sheet unless you are being directly asked to prove it.

1. Do Problem 14 in the text.
2. Do Problem 18 in the text.
3. Let X and Y be continuous random variables.
   (a) Prove that if $f_{xy}(x,y) = f_x(x)\, f_y(y)$ for all real x and y, then the random variables X and Y are independent. This result is also true if the condition holds except on a set of probability zero.
   (b) Prove that if X and Y are independent, then $f_{xy}(x,y) = f_x(x)\, f_y(y)$ at all points where $F_{xy}(x,y)$ is differentiable and $f_{xy}(x,y)$ is continuous.
4. Let X and Y be discrete random variables. Prove that if $p_{xy}(x,y) = p_x(x)\, p_y(y)$, then X and Y are independent.
5. Let $p_{xy}(x,y) = \frac{xy}{36}$ for $x = 1,2,3$ and $y = 1,2,3$, and zero otherwise.
   (a) What is $p_{y|x}(1|2)$?
   (b) What is $p_{x|y}(1|2)$?
   (c) Are x and y independent? Answer Yes or No and prove your answer.
6. Do Problem 20 in the text. Sketch the support first.
7. Problem 23 in the text is too challenging for a test or exam, but if you're up for it, use the Law of Total Probability, watch the limits of summation on n, change the variable of summation using k = n-x, and then apply the Binomial Theorem.
8. Do Problem 50 in the text.
9. Let X and Y be independent discrete random variables. Derive the convolution formula for the probability mass function of $Z = X + Y$. Use the Law of Total Probability.
10. Let X and Y be independent continuous random variables. Derive the convolution formula for the probability density function of $Z = X + Y$. Use Fubini's Theorem, which says you can always switch order of integration if what you are integrating is positive.

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11. Let $X \sim$ Poisson(λ_1) and $Y \sim$ Poisson(λ_2) be independent. Using the convolution formula, find the probability mass function of $Z = X + Y$ and identify it by name.

12. Let $X \sim$ Binomial(n_1, p) and $Y \sim$ Binomial(n_2, p) be independent. Using the convolution formula, find the probability mass function of $Z = X + Y$ and identify it by name.

13. Do Problem 43 in the text. Use the convolution formula. Consider the cases $0 < z \leq 1$ and $1 < z \leq 2$ separately. You need to pay close attention to the limits of integration, which are different for the two cases. If $f(x)$ is the density of the uniform $(0, 1)$ distribution, for what values of x is $f(z - x)$ non-zero? You will see that the density of $X = X + Y$ is triangular.

14. Let X and Y be independent Gamma random variables with parameters α, and $\lambda = 1$. Find the probability density function of $Z = X + Y$ and identify it by name.

    Once you apply the convolution formula, you are looking at a difficult integral. What you are trying to integrate looks a bit like a beta density; see the formula sheet. The change of variables $u = \frac{x}{z}$ gets you closer. Multiply and divide by the right quantity and you have it.

15. Do Problem 48 in the text, except let λ_1 and λ_2 have a single value λ. Use the Jacobian method. Identify the distribution of $T_1 + T_2$ by name.

16. Let X_1 and X_2 be independent standard normal random variables. Find the probability density function of $Y_1 = X_1/X_2$.

17. Do Problem 42a in the text. Use the Law of Total Probability and differentiate.

18. Here is another way to obtain the double exponential density of Question 17. Let X_1 and X_2 be independent exponential random variables with parameter λ, and let $Y_1 = X_1 - X_2$. Find the (marginal) probability density of Y_1. It's essential and a bit challenging to sketch the region where the joint density of Y_1 and Y_2 is non-zero.

19. Show that the normal density integrates to one. The formula $dx\,dy = r\,dr\,d\theta$ will be on the formula sheet.

20. Do Problem 61 in the text.

21. Do Problem 62 in the text.

22. Let $X_1, \ldots, X_n$ be independent random variables with probability density function $f(x) = e^{-x}$ for $x \geq 0$. Let $Y = \min(X_1, \ldots, X_n)$. Find the density $f_y(y)$.

23. Let $X_1, \ldots, X_n$ be independent Uniform $(0, \theta)$ random variables, and let $Y = \max(X_1, \ldots, X_n)$. Find the density $f_y(y)$.
