STA 256f18 Assignment Six[1]

Please read Sections 3.2 and 3.3 in Chapter 3 of the textbook and look over your lecture notes. These homework problems are not to be handed in. They are preparation for Term Test 2 and the final exam. All textbook problems are from Chapter Three. Use the formula sheet to do the problems. You will have a copy of Table 2 from Appendix B in the text. On tests and the final exam, you may use anything on the formula sheet unless you are being directly asked to prove it.

1. Do Problem 1a in the text.
2. For the joint distribution of Problem 1, give
   (a) $F_{xy}(3,2)$ The answer is a number.
   (b) $F_{xy}(4,4)$ The answer is a number.
   (c) $F_{xy}(0,0)$ The answer is a number.
   (d) $F_{xy}(2,1.5)$ The answer is a number.
   (e) $F_{xy}(3.5,7)$ The answer is a number.
   (f) $F_x(x)$. Your answer must apply to all real x.
3. Let $p_{xy}(x,y) = c(x+y)$ for $x = 1,2,3$, $y = 1,2$, and zero otherwise.
   (a) Find the constant c. The answer is a number.
   (b) What is $p_x(2)$? The answer is a number.
   (c) What is $p_y(1)$? The answer is a number.
   (d) What is $F_x(2.5)$? The answer is a number.
4. Do Problem 2 in the text.
5. Do Problem 3 in the text.
6. Do Problem 6 in the text. Assume $a > 0$ and $b > 0$. It helps to sketch the ellipse. The area of the ellipse is πab, a fact you may use without proof. If you are interested in the derivation of the formula for area, note that $2\int_{-a}^{a} \sqrt{a^2+x^2}\,dx$ is the area of a circle with radius a.
7. Do Problem 7 in the text.

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8. Do Problem 8ab in the text.

9. Do Problem 9a in the text.

10. Let X and Y be continuous random variables. Show $\lim_{x\to\infty} F_{xy}(x,y)) = F_y(y)$. Use Fubini's Theorem, which says you can always switch order of integration as long as what you are integrating is non-negative. Don't just move limits through integrals.

11. The continuous random variables X and Y have joint cumulative distribution function

$$F_{xy}(x,y) = \begin{cases} x^2(1-e^{-3y}) & \text{for } 0 \leq x \leq 1 \text{ and } y \geq 0 \\ 1-e^{-3y} & \text{for } x > 1 \text{ and } y \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

    (a) What is $F_{xy}(\frac{1}{2}, 3)$?
    (b) What is $F_{xy}(2, 1)$?
    (c) What is $F_{xy}(-1, 3)$?
    (d) What is $f_{xy}(x, y)$?
    (e) Obtain $f_y(y)$ by integrating out x.
    (f) Obtain $F_y(y)$ by taking limits.
    (g) Obtain $f_y(y)$ by differentiation.

12. Do Problem 10ab in the text. This problem requires too much geometric intuition to appear on a test or exam, but it's a valuable exercise anyway because it requires you to pay attention to limits of integration. The volume of a sphere is $\frac{4}{3}\pi r^3$. Don't hesitate to use your knowledge that the area of a circle is πr^2.

13. Do Problem 12ab in the text.

14. Do Problem 15abc in the text. Make sure you do 10b first. Part (b) of this question says sketch the joint density, but they must mean sketch the region where the joint density is non-zero.

15. Do Problem 17ab in the text.

16. Do Problem 18abc in the text. In addition, find $P(X + Y > 1)$.
