

STA 256f18 Assignment Two¹

Please read Sections 1.1-1.4 (pages 1-15) in the textbook and look over your lecture notes. These homework problems are not to be handed in. They are preparation for Term Test 1 and the final exam. Some of the questions are too easy to be on the test or exam, but they are good preparation for the real problems. All textbook problems are from Chapter One.

1. Do Problem 1 in the text. Assuming the coin is fair, also give the probabilities.
2. Do Problem 4 in the text.
3. Make Venn diagrams to illustrate the distributive laws:
 - (a) $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$
 - (b) $(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$
4. Prove $P(A^c) = 1 - P(A)$. Use the axioms of probability and the tabular format illustrated in lecture.
5. Prove $P(\emptyset) = 0$. Use the result from Problem 4, the axioms of probability and the tabular format illustrated in lecture.
6. Prove that if $A \subseteq B$ then $P(A) \leq P(B)$. Use the axioms of probability and the tabular format illustrated in lecture.
7. Prove the Addition Law: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. Use the axioms of probability and the tabular format illustrated in lecture.
8. To get ready for university, 50% of entering students purchase a new computer (or their parents buy it), and 40% purchase a new phone. If 30% get both, what percent of entering students get neither a new computer nor a new phone?
9. Do Problem 7 in the text. Use the tabular format illustrated in lecture. Bonferroni's inequality usually refers to the result in Problem 8, not 7. Use facts from the formula sheet.
10. Do Problem 8 in the text. Use the formula sheet, induction, and the tabular format illustrated in lecture.
11. Extend Problem 8 in the text to an infinite collection of sets. Use the formula sheet and the tabular format illustrated in lecture. Hint: Let $B_1 = A_1$, $B_2 = A_2 \cap A_1^c$, $B_3 = A_3 \cap (A_1 \cup A_2)^c$, and so on.
12. Do Problem 9 in the text.
13. Do Problems 25 and 15 in the text.
14. If eight children are standing in line,
 - (a) In how many orders can they stand?

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- (b) In how many orders can they stand if two friends insist on being together?
 - (c) Suppose there are four boys and four girls. In how many orders can they stand if the boys stay together and the girls stay together?
 - (d) If the children line up completely at random, what is the probability that the four boys are together and the four girls are together?
15. Do Problem 20 in the text.
 16. If there are 20 people in a club,
 - (a) In how many ways can a President, Vice-president and Treasurer be selected?
 - (b) In how many ways can three members be selected to go to a meeting?
 17. Do Problem 23 in the text.
 18. Do Problem 38 in the text.
 19. Do Problem 42 in the text.
 20. Do Problem 14 in the text. The answer has 29 digits, so stop simplifying after you arrive at a multinomial coefficient.
 21. A jar contains 10 red balls and 20 blue balls. If you sample 5 balls randomly without replacement, what is the probability of
 - (a) All blue?
 - (b) Two red and three blue?
 - (c) At least one red?
 - (d) Obtaining j red balls, $j = 0, \dots, 5$? Give a single formula. Don't simplify.
 22. Do Problem 11 in the text.
 23. Do Problem 17 in the text, except don't graph it. Give a formula instead. Let d denote the number of defectives. Clearly the probability of acceptance is zero if $d > 96$. Give a formula for the probability of acceptance as a function of d , for $d \leq 96$.
 24. Suppose that a room contains n people. What is the probability that at least two of them have a common birthday? Assume a year of 365 days, and that the chances of being born on all days are equal.
 25. Under the assumptions of Problem 24, how many people must you ask to have a 50-50 chance of finding someone whose birthday is the same as yours?

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<http://www.utstat.toronto.edu/~brunner/oldclass/256f18>