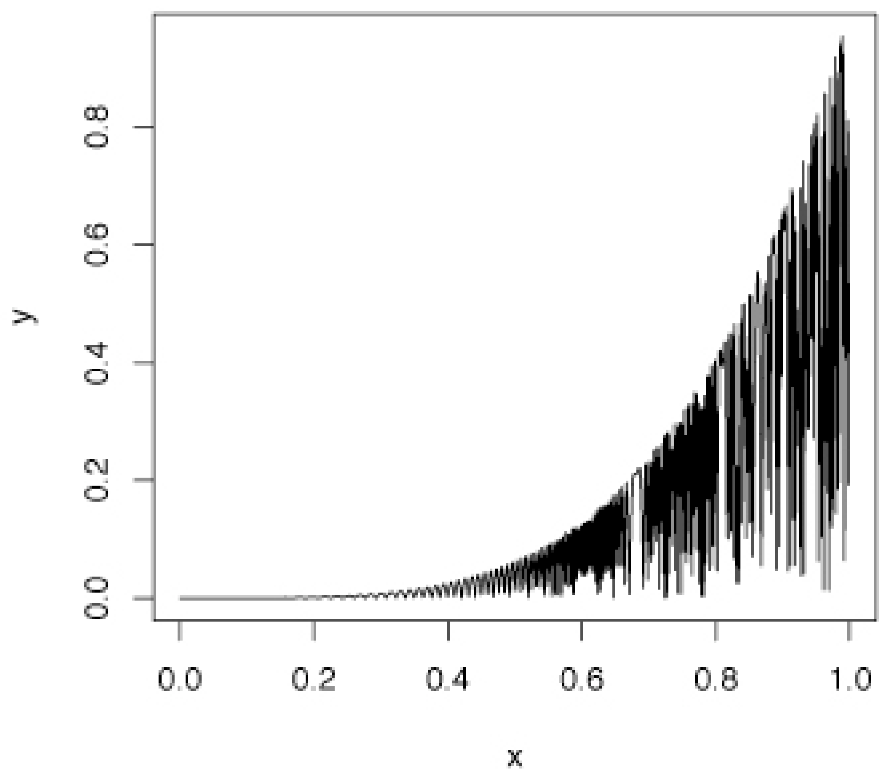
Monte Carlo Integration

$$\int_0^1 x^4 |\sin(100\tan(\cos(\frac{1}{1-x})))|\, dx$$

```
> rm(list=ls())
> g <- function(x)
+     {g = x^4*abs(sin(100*tan(cos(1/(1-x))))); g }
> x = seq(from=0.001,to=0.999,by=0.001)
> y = g(x)
> plot(x,y,type='l')
```



```
> # Monte Carlo Approximation of the integral from 0 to 1
> M = 10000; set.seed(4444); alpha= 0.001; z = qnorm(1-alpha/2)
> x = runif(M); y = g(x); area1 = mean(y)
> s1 = sqrt(var(y)); marginerr1 = s1/M*z; marginerr1
[1] 6.601693e-05
> cat("Estimate of integral is ",area1,"\n")
Estimate of integral is  0.1281166
> cat(100*(1-alpha),"percent CI from ",area1-marginerr1," to
",area1+marginerr1,"\n")
99.9 percent CI from  0.1280505  to  0.1281826
```

Numerical integration with Mathematica

```
/dos/brunner > math
Mathematica 6.0 for Sun Solaris SPARC (64-bit)
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In[1]:= f := x^4 Abs[Sin[100*Tan[Cos[1/(1-x)]]]]

In[2]:= NIntegrate[f,{x,0,1}]


NIntegrate::slwcon:
   Numerical integration converging too slowly; suspect one of the following:
    singularity, value of the integration is 0, highly oscillatory integrand,
    or WorkingPrecision too small.

NIntegrate::ncvb:
   NIntegrate failed to converge to prescribed accuracy after 9
     recursive bisections in x near {x} =
    {0.991925573252933004061993038646960 4<<13>>288818359375000000}
    . NIntegrate obtained 0.129104 and 0.00471027
     for the integral and error estimates.

Out[2]= 0.129104


> 0.129104 - 0.00471027
[1] 0.1243937
> 0.129104 + 0.00471027
[1] 0.1338143

> # Try a bigger NC sample size
> M = 100000; set.seed(4444); alpha= 0.001; z = qnorm(1-alpha/2)
> x = runif(M); y = g(x); area1 = mean(y)
> s1 = sqrt(var(y)); marginerr1 = s1/M*z; marginerr1
[1] 6.556855e-06
> cat("Estimate of integral is ",area1,"\n")
Estimate of integral is  0.1285584
> cat(100*(1-alpha),"percent CI from ",area1-marginerr1," to
",area1+marginerr1,"\n")
99.9 percent CI from  0.1285519  to  0.1285650
>
```