Chi-Square Power Examples

Discrete random variable with four categories - Test H_0: $\pi_1 = \pi_2$

True values: $\pi_1 = 0.15$, $\pi_2 = 0.25$, $\pi_3 = 0.15$, $\pi_4 = 0.45$

```
> rm(list=ls())
> n = 100; pi1 = 0.15; pi2 = 0.25; pi3 = 0.15 # So pi4 = 0.45
> critval = qchisq(0.95,2)
> # Likelihood Ratio Test (Agresti's Categorical Data Analysis)
> Pi = c(pi1,pi2,pi3,1-pi1-pi2-pi3)
> mpi = mean(c(pi1,pi2,pi3)) # mean pi under H0 (MLE looks like this)
> Pi0 = c(mpi,mpi,mpi,1-3*mpi)
> phiLR = 2*n * sum(Pi*log(Pi/Pi0))
> LRpow = 1-pchisq(critval,2,phiLR)
> cat("\nLR Test: phi = ",phiLR," Power = ",LRpow,"\n\n")

LR Test: phi =  3.467505  Power =  0.3659365

>
> nsim = 10000; set.seed(4444)
> nsig = 0
> for(i in 1:nsim)
+     {
+     pihat = as.vector(rmultinom(1,n,Pi)/n)
+     # LR test
+     mpihat = mean(pihat[1:3])
+     pihat0 = c(mpihat,mpihat,mpihat,1-3*mpihat)
+     G = 2*n * sum(pihat*log(pihat/pihat0))
+     if(G>critval) nsig = nsig+1
+     } # End simulation loop
> LRsimpow = nsig/nsim
> cat("\nLikelihood Ratio Test: ",LRpow," by ncp, ",LRsimpow,
+ " by simulation \n")

Likelihood Ratio Test:  0.3659365  by ncp,  0.3721  by simulation
```

Wald Test $W_2 = n\,(\mathbf{C}\widehat{\theta} - \mathbf{h})'(\mathbf{C}\mathbf{I}(\widehat{\theta})^{-1}\mathbf{C}')^{-1}(\mathbf{C}\widehat{\theta} - \mathbf{h})$

$$I(\boldsymbol{\pi})^{-1} = \begin{pmatrix} \pi_1(1-\pi_1) & -\pi_1\pi_2 & -\pi_1\pi_3 \\ -\pi_1\pi_2 & \pi_2(1-\pi_2) & -\pi_2\pi_3 \\ -\pi_1\pi_3 & -\pi_2\pi_3 & \pi_3(1-\pi_3) \end{pmatrix}$$

```
> # Wald
> Vmat <- function(p1,p2,p3)
+     {
+     Vmat = diag(c(p1*(1-p1),p2*(1-p2),p3*(1-p3)))
+     Vmat[1,2] = Vmat[2,1] = -p1*p2
+     Vmat[1,3] = Vmat[3,1] = -p1*p3
+     Vmat[2,3] = Vmat[3,2] = -p2*p3
+     Vmat
+     } # End function Vmat
> V = Vmat(pi1,pi2,pi3); V
        [,1]    [,2]    [,3]
[1,]  0.1275 -0.0375 -0.0225
[2,] -0.0375  0.1875 -0.0375
[3,] -0.0225 -0.0375  0.1275
> CC = rbind(c(-1,0,1),c(0,-1,1))
> kore <- solve(CC%*%V%*%t(CC))
> effect = rbind(pi3-pi1,pi3-pi2); effect
     [,1]
[1,]  0.0
[2,] -0.1
> phiW = n * t(effect)%*%kore%*%effect; phiW = as.numeric(phiW)
> Wpow = 1-pchisq(critval,2,phiW)
> cat("\nWald Test: phi = ",phiW," Power = ",Wpow,"\n\n")

Wald Test: phi =  3.174603  Power =  0.3381904

>
> nsim = 10000; set.seed(4444)
> nsigW = 0
> for(i in 1:nsim)
+     {
+     pihat = as.vector(rmultinom(1,n,Pi)/n)
+     Vhat = Vmat(pihat[1],pihat[2],pihat[3])
+     kore2 <- solve(CC%*%Vhat%*%t(CC))
+     effhat = CC%*%pihat[1:3]
+     W = n * t(effhat)%*%kore2%*%effhat
+     if(W>critval) nsigW = nsigW+1
+     } # End simulation loop
> Wsimpow = nsigW/nsim
> cat("Wald: ",Wpow," by ncp, ",Wsimpow," by simulation \n")
Wald:  0.3381904  by ncp,  0.3705  by simulation
> cat("LR  : ",LRpow," by ncp, ",LRsimpow," by simulation \n")
LR  :  0.3659365  by ncp,  0.3721  by simulation
```

```
> # Now use the general LR test way to assess power of a test of
> # H0: theta1=theta2=1/4 for the 4-category multinomial
>
> rm(list=ls())
> n = 100; critval = qchisq(0.95,2)
> pi = c(0.15, 0.35, 0.3, 0.2); h1 = 0.25; h2 = 0.25
> piM3 = (1-h1-h2) * pi[3]/(1-pi[1]-pi[2])
> piM = c(h1,h2,piM3,1-h1-h2-piM3)
> phi = 2*n * sum(pi*log(pi/piM))
> Apower = 1-pchisq(critval,2,phi) # 0.730466
> cat("\nPower with Agresti's formula = ",Apower,"\n\n")

Power with Agresti's formula =  0.730466

> Info <- function(p1,p2,p3)
+     {
+     Vmat = diag(c(p1*(1-p1),p2*(1-p2),p3*(1-p3)))
+     Vmat[1,2] = Vmat[2,1] = -p1*p2
+     Vmat[1,3] = Vmat[3,1] = -p1*p3
+     Vmat[2,3] = Vmat[3,2] = -p2*p3
+     Info = solve(Vmat)
+     Info = Info[1:2,1:2]
+     Info
+     } # End function Info
> Ir = Info(pi[1],pi[2],pi[3])
> truth = cbind(c(pi[1]-h1,pi[2]-h2))
> phi2 = n * t(truth) %*% Ir %*% truth
> Wpower = 1-pchisq(critval,2,phi2) #
> cat("\nPower with Wald's formula = ",Wpower,"\n\n")

Power with Wald's formula =  0.7951132

> ############################################################
> nsim = 10000; set.seed(4444)
> nsig = 0
> for(i in 1:nsim)
+     {
+     x <- as.numeric(rmultinom(1,n,pi))
+     pi03 <- (1-h1-h2) * x[3]/(x[3]+x[4])
+     pihat0 <- c(h1,h2,pi03,1-h1-h2-pi03); pihat0
+     fe = pihat0*n
+     G = 2*sum(x*log(x/fe))
+     if(G>critval) nsig = nsig+1
+     } # End simulation loop
> cat("Power by simulation with ",nsim," simulations = ",nsig/nsim,"\n\n")
Power by simulation with  10000  simulations =  0.7389

> cat("\nPower with Agresti's formula = ",Apower,"\n\n")

Power with Agresti's formula =  0.730466

> cat("\nPower with Wald's formula = ",Wpower,"\n\n")

Power with Wald's formula =  0.7951132
```