STA 2201 S 2011 Assignment 6

Let $\mathbf{X}_1, \ldots, \mathbf{X}_n$ be a random sample from a k-dimensional multivariate normal distribution with expected value $\boldsymbol{\mu}$ and variance-covariance matrix $\boldsymbol{\Sigma}$. The interest is in a single test test for whether the "variables" (meaning the elements of $\mathbf{X}_i$) are related to one another, so the null hypothesis is that $\boldsymbol{\Sigma}$ is diagonal.

1. Derive the large-sample likelihood ratio test for this problem. It takes a nice simple form. Show your work.

2. Using the data file `Assignment6.data`, test $H_0 : \sigma_{i,j} = 0$ for $i \neq j$ at $\alpha = 0.05$. There is a link to the data file from the course home page as well as from this document. Your computer output should include the value of the test statistic, the degrees of freedom, and the p-value. Do you reject the null hypothesis?

3. Suppose that the true variance covariance matrix is

$$
\boldsymbol{\Sigma} = \begin{bmatrix}
1.42 & 0.42 & 0.42 & 0.00 & 0.00 \\
0.42 & 1.42 & 0.42 & 0.00 & 0.00 \\
0.42 & 0.42 & 1.42 & 0.00 & 0.00 \\
0.00 & 0.00 & 0.00 & 1.18 & 0.18 \\
0.00 & 0.00 & 0.00 & 0.18 & 1.18
\end{bmatrix}.
$$

   What is the power of the test at $n = 50$? Please accompany your estimate with a 99% confidence interval. Display computer input and output.

4. What minimum sample size would be required for a power of at least 0.80 against this alternative? Give an estimated power and a 99% confidence interval for the estimated power along with your final answer. Display computer input and output.