Limits Assignment 1

Solve these problems using only the definition of a limit and rules of elementary algebra (logs and stuff like that are okay, but no calculus).

1. Prove $\frac{3n + 1}{4n + 17} \to 3/4$.

2. Let a, b, c, and d be non-zero real numbers. Find $\lim_{n\to\infty} \frac{a\,n + b}{c\,n + d}$ and prove your answer in general, *not* for specific values of a, b, c, and d.

3. Show $\frac{32}{n^p} \to 0$, where p is an arbitrary positive number. Do not do it for a particular numerical value of p. This is always the case, so I'll stop saying it now.

4. Prove that if $a_n = a$ for each n, $a_n \to a$.

5. Prove that $\lim_{n\to\infty} x^n = 0$ if $|x| < 1$.

6. Prove that if $a_n \to 4$, then $3a_n \to 12$.

7. Prove $\frac{1}{2^{\sqrt{n}}} \to 0$.

8. Let $a_n \to A$ and $b_n \to B$. Show $a_n b_n \to AB$. Hint: $a_n b_n - AB = a_n(b_n - B) + B(a_n - A)$

9. Prove $\frac{\sqrt{n}}{n + 14} \to 0$.

10. A set of real numbers is said to be *bounded* if there exist real numbers A and B such that each number in the set is between A and B. Prove that if $a_n \to a \in \mathbb{R}$, the entire sequence $\{a_n\}$ is bounded.

11. Assume $a_n \to x$ and $x>y$. Show $\exists N_1 \in \mathbb{R} \ni$ if $n > N_1$, $a_n > x - (x-y)/3$.

12. Assume $a_n > x - (x-y)/3$ and $a_n < y + (x-y)/3$. Show that $x < y$.

Note that **you must know the definition of a limit**. It could be asked, and it could be worth a lot of points.