Homework 10: Quiz Nov. 25

You may bring the convergence handout.

1. Let $X_1, X_2, \ldots, X_n$ be i.i.d. random variables whose distribution depends on a parameter θ. The statistic $T_n = T_n(X_1, X_2, \ldots, X_n)$ is said to be *consistent* for θ if $T_n \xrightarrow{P} \theta$. It is *unbiased* if $E[T_n] = \theta$. Suppose $X_1, X_2, \ldots, X_n$ are continuous uniform random variables on the interval $(0, \theta)$.
   (a) Let $T_n = 2\overline{X}_n$. Is T_n unbiased? Is it consistent? Show your work.
   (b) Let Y_n be the maximum of $X_1, X_2, \ldots, X_n$. Is Y_n unbiased? Is it consistent? Show your work.
   (c) Define a statistic W_n that is a one-to-one function of Y_n and is unbiased. Is it also consistent? Show your work.

2. Let $X_1, X_2, \ldots, X_n$ be i.i.d. random variables with expected value μ and variance σ^2. Show $\frac{\sqrt{n}(\overline{X}_n - \mu)}{S_n} \xrightarrow{d} N(0,1)$, where S_n is the sample standard deviation. Don't do it from scratch; use the convergence handout.

3. Let $X_1, X_2, \ldots, X_n$ be i.i.d. random variables with expected value μ and variance σ^2. Define $Y_n = \sqrt{n}(\overline{X}_n - \mu)$. What is the asymptotic distribution of e^{Y_n}? What is $\lim_{n\to\infty} P(e^{Y_n} < 1)$?

4. Let $\sqrt{n}(T_n - \theta) \xrightarrow{d} T$. Prove $T_n \xrightarrow{P} \theta$.

5. Let $T_n \xrightarrow{d} T$. Prove $\frac{T_n}{\log n} \xrightarrow{P} 0$.

6. Using Stirling's formula, show that a t-distribution with n degrees of freedom converges to a standard normal. Stirling's formula for gamma functions is just what you might think.

7. Let $X_1, X_2, \ldots, X_n$ be a random sample from a distribution with expected value μ and variance σ^2. Let $T_n = \frac{\sum_{i=1}^n X_i}{n - 237.6}$. Show $\sqrt{n}(T_n - \mu)$ has a limiting normal distribuion. What is the variance of the limiting distribution?

8. Let $X_1, X_2, \ldots, X_n$ be i.i.d. random variables with finite fourth moment; denote their commom expected value by μ and their variance by σ^2. Show $\sqrt{n}(S_n^2 - \sigma^2)$ (where S_n^2 is the common sample variance) converges in distribution to a normal random variable. What is the variance of this normal target? Don't bother to simplify. Hint: Start by letting $Y_i = (X_i - \mu)^2$; later, seek asymptotic equivalence.