Hint for Problem 4.41c

This problem is a good example of why it is better to deal with the multivariate normal distribution using matrix notation. If you try to do this problem with either a one-variable or two-variable Jacobian approach, it's a huge mess. However, try defining

$$Z_1 = \frac{\left(\frac{X_1-\mu_1}{\sigma_1} - \rho\left(\frac{X_2-\mu_2}{\sigma_2}\right)\right)}{\sqrt{1-\rho^2}} \text{ and } Z_2 = \frac{X_2-\mu_2}{\sigma_2}.$$

Now a two-variable Jacobian exercise shows that Z_1 and Z_2 are *independent* standard normal. Writing

$$X_1 = \sigma_1(Z_1\sqrt{1-\rho^2} + \rho Z_2) + \mu_1 \text{ and } X_2 = \sigma_2 Z_2 + \mu_2,$$

re-express $Y = aX_1 + bX_2$ as $Y = \alpha Z_1 + \beta Z_2 + \mu$. You can get the density of Y directly using a Jacobian without too much pain, but it's easier to write the moment-generating function of Y.