The BMI Health Study

```
> rm(list=ls()); options(scipen=999)
> # install.packages("lavaan", dependencies = TRUE) # Only need to do this once
> library(lavaan)
This is lavaan 0.6-3
lavaan is BETA software! Please report any bugs.
>
> bmidata =
read.table("http://www.utstat.toronto.edu/~brunner/openSEM/data/bmi.data.txt")
> head(bmidata)
  age1 bmi1 fat1 cholest1 diastol1 age2 bmi2 fat2 cholest2 diastol2
1   63 24.5 16.5    195.4       38   60 23.9 20.1    203.5       66
2   42 13.0  1.9    184.3       86   44 14.8  2.6    197.3       78
3   32 22.5 14.6    354.1      104   33 21.7 20.4    374.3       73
4   59 25.5 19.0    214.6       93   58 28.5 20.0    203.7      106
5   45 26.5 17.8    324.8       97   43 25.0 12.3    329.7       92
6   31 19.4 17.1    280.7       92   42 19.9 19.9    276.7       87
>
> # Ordinary linear regression. Average the measurements
>
> attach(bmidata)
> age = (age1+age2)/2; bmi = (bmi1+bmi2)/2; fat = (fat1+fat2)/2
> cholest = (cholest1+cholest2)/2; diastol = (diastol1+diastol2)/2
>
> fullmod = lm( cbind(cholest,diastol) ~ age + fat + bmi)
> restrictedmod = update(fullmod, . ~ . - bmi) # Remove the variable(s) being tested
> anova(fullmod,restrictedmod) # Gives multivariate test.
Analysis of Variance Table

Model 1: cbind(cholest, diastol) ~ age + fat + bmi
Model 2: cbind(cholest, diastol) ~ age + fat
  Res.Df Df Gen.var.  Pillai approx F num Df den Df    Pr(>F)
1    496      591.89
2    497  1   599.36 0.02869   7.3106      2    495 0.0007431 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
>
> summary(fullmod) # Two sets of univariate output
```

```
Response cholest :

Call:
lm(formula = cholest ~ age + fat + bmi)

Residuals:
     Min       1Q   Median       3Q      Max
-148.550  -34.243    2.626   33.661  165.582

Coefficients:
            Estimate Std. Error t value             Pr(>|t|)
(Intercept) 220.0610    21.0109  10.474 < 0.0000000000000002 ***
age          -0.2714     0.2002  -1.356              0.17578
fat           2.2334     0.5792   3.856              0.00013 ***
bmi           0.5164     1.0154   0.509              0.61128
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 52.43 on 496 degrees of freedom
Multiple R-squared:  0.09701, Adjusted R-squared:  0.09155
F-statistic: 17.76 on 3 and 496 DF,  p-value: 0.00000000005762


Response diastol :

Call:
lm(formula = diastol ~ age + fat + bmi)

Residuals:
    Min      1Q  Median      3Q     Max
-44.841  -7.140  -0.408   7.612  41.377

Coefficients:
            Estimate Std. Error t value             Pr(>|t|)
(Intercept) 49.69194    4.52512  10.981 < 0.0000000000000002 ***
age          0.12648    0.04311   2.934             0.003504 **
fat          0.64056    0.12474   5.135          0.000000406 ***
bmi          0.82627    0.21869   3.778             0.000177 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 11.29 on 496 degrees of freedom
Multiple R-squared:  0.3333,  Adjusted R-squared:  0.3293
F-statistic: 82.67 on 3 and 496 DF,  p-value: < 0.00000000000000022
```

```
> bmimodel1 =
+     ###########################################################
+     # Latent variable model
+     # ---------------------
+    'Fcholest ~ beta11*Fage + beta12*Fbmi + beta13*Ffat
+     Fdiastol ~ beta21*Fage + beta22*Fbmi + beta23*Ffat
+     #
+     # Measurement model
+     # -----------------
+     Fage =~ 1*age1 + 1*age2
+     Fbmi =~ 1*bmi1 + 1*bmi2
+     Ffat =~ 1*fat1 +1*fat2
+     Fcholest =~ 1*cholest1 + 1*cholest2
+     Fdiastol =~ 1*diastol1 + 1*diastol2
+     #
+     # Variances and covariances
+     # -------------------------
+     # Of latent explanatory variables
+     Fage ~~ phi11*Fage; Fage ~~ phi12*Fbmi; Fage ~~ phi13*Ffat
+                         Fbmi ~~ phi22*Fbmi; Fbmi ~~ phi23*Ffat
+                                            Ffat ~~ phi33*Ffat
+     # Of error terms in latent the regression (epsilon_ij)
+     Fcholest ~~ psi11*Fcholest; Fcholest ~~ psi12*Fdiastol
+                                 Fdiastol ~~ psi22*Fdiastol
+    # Of measurement errors (e_ijk) for measurement set 1
+    age1 ~~ w111*age1; age1 ~~ w112*bmi1; age1 ~~ w113*fat1;
+    age1 ~~ w114*cholest1; age1 ~~ w115*diastol1
+            bmi1 ~~ w122*bmi1; bmi1 ~~ w123*fat1; bmi1 ~~ w124*cholest1; bmi1 ~~ w125*diastol1
+                    fat1 ~~ w133*fat1; fat1 ~~ w134*cholest1; fat1 ~~ w135*diastol1
+                            cholest1 ~~ w144*cholest1; cholest1 ~~ w145*diastol1
+                                                       diastol1 ~~ w155*diastol1
+    # Of measurement errors (e_ijk) for measurement set 2
+    age2 ~~ w211*age2; age2 ~~ w212*bmi2; age2 ~~ w213*fat2;
+    age2 ~~ w214*cholest2; age2 ~~ w215*diastol2
+            bmi2 ~~ w222*bmi2; bmi2 ~~ w223*fat2; bmi2 ~~ w224*cholest2; bmi2 ~~ w225*diastol2
+                    fat2 ~~ w233*fat2; fat2 ~~ w234*cholest2; fat2 ~~ w235*diastol2
+                            cholest2 ~~ w244*cholest2; cholest2 ~~ w245*diastol2
+                                                       diastol2 ~~ w255*diastol2
+    ' ################## End of bmimodel1 ##################
>
> fit1 = lavaan(bmimodel1, data=bmidata)
Warning message:
In lavaan(bmimodel1, data = bmidata) :
  lavaan WARNING: the optimizer warns that a solution has NOT been found!
>
>

> # The output from summary is very large. There are 45 parameters.
> show(fit1)
lavaan 0.6-3 did NOT end normally after 5685 iterations
** WARNING ** Estimates below are most likely unreliable

  Optimization method                           NLMINB
  Number of free parameters                         45

  Number of observations                           500

  Estimator                                         ML
  Model Fit Test Statistic                          NA
  Degrees of freedom                                NA
  P-value                                           NA
```

```
> parTable(fit1)
   id      lhs op      rhs user block group free ustart exo  label plabel    start
1   1 Fcholest  ~     Fage    1     1     1    1     NA   0 beta11   .p1.    0.000
2   2 Fcholest  ~     Fbmi    1     1     1    2     NA   0 beta12   .p2.    0.000
3   3 Fcholest  ~     Ffat    1     1     1    3     NA   0 beta13   .p3.    0.000
4   4 Fdiastol  ~     Fage    1     1     1    4     NA   0 beta21   .p4.    0.000
5   5 Fdiastol  ~     Fbmi    1     1     1    5     NA   0 beta22   .p5.    0.000
6   6 Fdiastol  ~     Ffat    1     1     1    6     NA   0 beta23   .p6.    0.000
7   7     Fage =~     age1    1     1     1    0      1   0           .p7.    1.000
8   8     Fage =~     age2    1     1     1    0      1   0           .p8.    1.000
9   9     Fbmi =~     bmi1    1     1     1    0      1   0           .p9.    1.000
10 10     Fbmi =~     bmi2    1     1     1    0      1   0          .p10.    1.000
11 11     Ffat =~     fat1    1     1     1    0      1   0          .p11.    1.000
12 12     Ffat =~     fat2    1     1     1    0      1   0          .p12.    1.000
13 13 Fcholest =~ cholest1    1     1     1    0      1   0          .p13.    1.000
14 14 Fcholest =~ cholest2    1     1     1    0      1   0          .p14.    1.000
15 15 Fdiastol =~ diastol1    1     1     1    0      1   0          .p15.    1.000
16 16 Fdiastol =~ diastol2    1     1     1    0      1   0          .p16.    1.000
17 17     Fage ~~     Fage    1     1     1    7     NA   0  phi11   .p17.    0.050
18 18     Fage ~~     Fbmi    1     1     1    8     NA   0  phi12   .p18.    0.000
19 19     Fage ~~     Ffat    1     1     1    9     NA   0  phi13   .p19.    0.000
20 20     Fbmi ~~     Fbmi    1     1     1   10     NA   0  phi22   .p20.    0.050
21 21     Fbmi ~~     Ffat    1     1     1   11     NA   0  phi23   .p21.    0.000
22 22     Ffat ~~     Ffat    1     1     1   12     NA   0  phi33   .p22.    0.050
23 23 Fcholest ~~ Fcholest    1     1     1   13     NA   0  psi11   .p23.    0.050
24 24 Fcholest ~~ Fdiastol    1     1     1   14     NA   0  psi12   .p24.    0.000
25 25 Fdiastol ~~ Fdiastol    1     1     1   15     NA   0  psi22   .p25.    0.050
26 26     age1 ~~     age1    1     1     1   16     NA   0   w111   .p26.   83.763
27 27     age1 ~~     bmi1    1     1     1   17     NA   0   w112   .p27.    0.000
28 28     age1 ~~     fat1    1     1     1   18     NA   0   w113   .p28.    0.000
29 29     age1 ~~ cholest1    1     1     1   19     NA   0   w114   .p29.    0.000
30 30     age1 ~~ diastol1    1     1     1   20     NA   0   w115   .p30.    0.000
31 31     bmi1 ~~     bmi1    1     1     1   21     NA   0   w122   .p31.   10.925
32 32     bmi1 ~~     fat1    1     1     1   22     NA   0   w123   .p32.    0.000
33 33     bmi1 ~~ cholest1    1     1     1   23     NA   0   w124   .p33.    0.000
34 34     bmi1 ~~ diastol1    1     1     1   24     NA   0   w125   .p34.    0.000
35 35     fat1 ~~     fat1    1     1     1   25     NA   0   w133   .p35.   30.023
36 36     fat1 ~~ cholest1    1     1     1   26     NA   0   w134   .p36.    0.000
37 37     fat1 ~~ diastol1    1     1     1   27     NA   0   w135   .p37.    0.000
38 38 cholest1 ~~ cholest1    1     1     1   28     NA   0   w144   .p38. 1548.559
39 39 cholest1 ~~ diastol1    1     1     1   29     NA   0   w145   .p39.    0.000
40 40 diastol1 ~~ diastol1    1     1     1   30     NA   0   w155   .p40.  162.507
41 41     age2 ~~     age2    1     1     1   31     NA   0   w211   .p41.   76.888
42 42     age2 ~~     bmi2    1     1     1   32     NA   0   w212   .p42.    0.000
43 43     age2 ~~     fat2    1     1     1   33     NA   0   w213   .p43.    0.000
44 44     age2 ~~ cholest2    1     1     1   34     NA   0   w214   .p44.    0.000
45 45     age2 ~~ diastol2    1     1     1   35     NA   0   w215   .p45.    0.000
46 46     bmi2 ~~     bmi2    1     1     1   36     NA   0   w222   .p46.    7.156
47 47     bmi2 ~~     fat2    1     1     1   37     NA   0   w223   .p47.    0.000
48 48     bmi2 ~~ cholest2    1     1     1   38     NA   0   w224   .p48.    0.000
49 49     bmi2 ~~ diastol2    1     1     1   39     NA   0   w225   .p49.    0.000
50 50     fat2 ~~     fat2    1     1     1   40     NA   0   w233   .p50.   27.528
51 51     fat2 ~~ cholest2    1     1     1   41     NA   0   w234   .p51.    0.000
52 52     fat2 ~~ diastol2    1     1     1   42     NA   0   w235   .p52.    0.000
53 53 cholest2 ~~ cholest2    1     1     1   43     NA   0   w244   .p53. 1608.286
54 54 cholest2 ~~ diastol2    1     1     1   44     NA   0   w245   .p54.    0.000
55 55 diastol2 ~~ diastol2    1     1     1   45     NA   0   w255   .p55.   88.049
          est se
1      87.986 NA
2    1307.372 NA
3    -698.539 NA
4    -696.326 NA
5  -10306.408 NA
6    5531.568 NA
```

7	1.000	0
8	1.000	0
9	1.000	0
10	1.000	0
11	1.000	0
12	1.000	0
13	1.000	0
14	1.000	0
15	1.000	0
16	1.000	0
17	146.894	NA
18	3.027	NA
19	24.137	NA
20	11.534	NA
21	21.875	NA
22	43.806	NA
23	1439.256	NA
24	8682.195	NA
25	-68805.034	NA
26	18.784	NA
27	4.303	NA
28	0.968	NA
29	3.101	NA
30	12.518	NA
31	9.257	NA
32	7.088	NA
33	-0.995	NA
34	12.457	NA
35	17.494	NA
36	10.042	NA
37	-6.847	NA
38	199.263	NA
39	-1.087	NA
40	203.809	NA
41	8.187	NA
42	1.026	NA
43	-3.523	NA
44	-4.153	NA
45	5.971	NA
46	3.043	NA
47	-2.598	NA
48	-4.928	NA
49	7.475	NA
50	9.624	NA
51	-10.696	NA
52	-8.263	NA
53	347.391	NA
54	-14.307	NA
55	59.783	NA

```
> # The starting values are nicely within the parameter space, but Lavaan lacks the
sophisticated bag of tricks that SAS and other commercial software uses to pick
starting values. Obtain method of moments estimators from the identifiability
proof, and use those as starting values.
>
> # As I was doing this and comparing results with the values above, I finally
noticed the enormous negative estimated variance for diastol: psi22hat =
-68805.034. There was no warning. Run it again with bounds, and hope the search
bounces off the barrier into a better region of the parameter space.

> bmimodel1 =
+     ###########################################################
+     # Latent variable model
+     # ---------------------
+     'cholest ~ beta11*age + beta12*bmi + beta13*fat
+     diastol ~ beta21*age + beta22*bmi + beta23*fat
+     #
+     # Measurement model
+     # -----------------
+     age =~ 1*age1 + 1*age2
+     bmi =~ 1*bmi1 + 1*bmi2
+     fat =~ 1*fat1 +1*fat2
+     cholest =~ 1*cholest1 + 1*cholest2
+     diastol =~ 1*diastol1 + 1*diastol2
+     #
+     # Variances and covariances
+     # -------------------------
+     # Of latent explanatory variables
+     age ~~ phi11*age; age ~~ phi12*bmi; age ~~ phi13*fat
+                          bmi ~~ phi22*bmi; bmi ~~ phi23*fat
+                                             fat ~~ phi33*fat
+     # Of error terms in latent the regression (epsilon_ij)
+     cholest ~~ psi11*cholest; cholest ~~ psi12*diastol
+                                diastol ~~ psi22*diastol
+     # Of measurement errors (e_ijk) for measurement set 1
+     age1 ~~ w111*age1; age1 ~~ w112*bmi1; age1 ~~ w113*fat1;
+    age1 ~~ w114*cholest1; age1 ~~ w115*diastol1
+           bmi1 ~~ w122*bmi1; bmi1 ~~ w123*fat1; bmi1 ~~ w124*cholest1; bmi1 ~~ w125*diastol1
+                  fat1 ~~ w133*fat1; fat1 ~~ w134*cholest1; fat1 ~~ w135*diastol1
+                         cholest1 ~~ w144*cholest1; cholest1 ~~ w145*diastol1
+                                                    diastol1 ~~ w155*diastol1
+    # Of measurement errors (e_ijk) for measurement set 2
+    age2 ~~ w211*age2; age2 ~~ w212*bmi2; age2 ~~ w213*fat2;
+    age2 ~~ w214*cholest2; age2 ~~ w215*diastol2
+           bmi2 ~~ w222*bmi2; bmi2 ~~ w223*fat2; bmi2 ~~ w224*cholest2; bmi2 ~~ w225*diastol2
+                  fat2 ~~ w233*fat2; fat2 ~~ w234*cholest2; fat2 ~~ w235*diastol2
+                         cholest2 ~~ w244*cholest2; cholest2 ~~ w245*diastol2
+                                                    diastol2 ~~ w255*diastol2
+     # Bounds (Variances are positive)
+     # ------
+     phi11 > 0; phi22 > 0 ; phi33 > 0
+     psi11 > 0; psi22 > 0
+     w111 > 0; w122 > 0; w133 > 0; w144 > 0; w155 > 0;
+     w211 > 0; w222 > 0; w233 > 0; w244 > 0; w255 > 0
+     ' ################## End of bmimodel1 ##################
```

```
> fit1 = lavaan(bmimodel1, data=bmidata)
Warning messages:
1: In lav_model_vcov(lavmodel = lavmodel, lavsamplestats = lavsamplestats,  :
  lavaan WARNING:
    The variance-covariance matrix of the estimated parameters (vcov)
    does not appear to be positive definite! The smallest eigenvalue
    (= -7.544027e-07) is smaller than zero. This may be a symptom that
    the model is not identified.
2: In lav_object_post_check(object) :
  lavaan WARNING: some estimated lv variances are negative
>
>
>
> inspect(fit1,"cov.lv") # Can't see any
         age      bmi      fat      cholst   diastl
age      146.684
bmi        3.033   11.683
fat       24.483   21.894   43.455
cholest   21.599   65.440  120.970 2893.126
diastol   37.602   26.747   54.468  109.220  140.707
>
> summary(fit1)
lavaan 0.6-3 ended normally after 1245 iterations

  Optimization method                           NLMINB
  Number of free parameters                         45
  Number of inequality constraints                  15

  Number of observations                           500

  Estimator                                         ML
  Model Fit Test Statistic                     131.150
  Degrees of freedom                                10
  P-value (Chi-square)                           0.000

Parameter Estimates:

  Information                                 Expected
  Information saturated (h1) model          Structured
  Standard Errors                             Standard

Latent Variables:
                   Estimate   Std.Err  z-value  P(>|z|)
  age =~
    age1              1.000
    age2              1.000
  bmi =~
    bmi1              1.000
    bmi2              1.000
  fat =~
    fat1              1.000
    fat2              1.000
  cholest =~
    cholest1          1.000
    cholest2          1.000
  diastol =~
    diastol1          1.000
    diastol2          1.000
```

```
Regressions:
                   Estimate   Std.Err  z-value   P(>|z|)
  cholest ~
    age     (bt11)   138.684    1.239   111.888    0.000
    bmi     (bt12)  1986.200    0.245  8121.807    0.000
    fat     (bt13) -1076.064    0.366 -2937.006    0.000
  diastol ~
    age     (bt21)   -23.800    0.219  -108.712    0.000
    bmi     (bt22)  -340.740    0.602  -566.338    0.000
    fat     (bt23)   186.338    0.369   504.812    0.000

Covariances:
                   Estimate   Std.Err  z-value   P(>|z|)
  age ~~
    bmi     (ph12)     3.033    1.968     1.541    0.123
    fat     (ph13)    24.483    3.955     6.190    0.000
  bmi ~~
    fat     (ph23)    21.894    1.555    14.082    0.000
 .cholest ~~
   .diastol (ps12)   380.142    0.385   988.159    0.000
 .age1 ~~
   .bmi1    (w112)     4.501    0.922     4.883    0.000
   .fat1    (w113)     1.103    1.469     0.751    0.453
   .cholst1 (w114)     3.004    8.667     0.347    0.729
   .diastl1 (w115)    11.796    3.804     3.101    0.002
 .bmi1 ~~
   .fat1    (w123)     7.229    0.906     7.980    0.000
   .cholst1 (w124)    -1.336    3.580    -0.373    0.709
   .diastl1 (w125)    10.860    2.174     4.996    0.000
 .fat1 ~~
   .cholst1 (w134)    10.086    5.782     1.744    0.081
   .diastl1 (w135)    -1.710    3.340    -0.512    0.609
 .cholest1 ~~
   .diastl1 (w145)    -1.596    5.913    -0.270    0.787
 .age2 ~~
   .bmi2    (w212)     1.153    0.717     1.607    0.108
   .fat2    (w213)    -3.421    1.334    -2.565    0.010
   .cholst2 (w214)    -4.589    8.659    -0.530    0.596
   .diastl2 (w215)     1.333    2.624     0.508    0.611
 .bmi2 ~~
   .fat2    (w223)    -2.505    0.683    -3.666    0.000
   .cholst2 (w224)    -5.272    3.047    -1.730    0.084
   .diastl2 (w225)     2.462    1.351     1.822    0.068
 .fat2 ~~
   .cholst2 (w234)   -10.671    5.530    -1.930    0.054
   .diastl2 (w235)    -5.451    2.376    -2.294    0.022
 .cholest2 ~~
   .diastl2 (w245)    -8.625    8.577    -1.006    0.315
```

```
Variances:
                   Estimate   Std.Err  z-value   P(>|z|)
    age     (ph11)   146.684    9.494    15.450    0.000
    bmi     (ph22)    11.683    0.821    14.231    0.000
    fat     (ph33)    43.455    3.026    14.362    0.000
   .cholest (ps11)    91.104    0.053  1710.629    0.000
   .diastol (ps22)    -0.000
   .age1    (w111)    18.825    2.865     6.570    0.000
   .bmi1    (w122)     9.500    0.703    13.512    0.000
   .fat1    (w133)    17.210    1.589    10.833    0.000
   .cholst1 (w144)   199.838    0.485   412.035    0.000
   .diastl1 (w155)   198.322    1.343   147.715    0.000
   .age2    (w211)     8.377    2.653     3.158    0.002
   .bmi2    (w222)     3.260    0.466     6.998    0.000
   .fat2    (w233)     9.067    1.514     5.990    0.000
   .cholst2 (w244)   347.816    0.164  2120.241    0.000
   .diastl2 (w255)    38.493    6.501     5.921    0.000

Constraints:
                                                |Slack|
    phi11 - 0                                   146.684
    phi22 - 0                                    11.683
    phi33 - 0                                    43.455
    psi11 - 0                                    91.104
    psi22 - 0                                     0.000
    w111 - 0                                     18.825
    w122 - 0                                      9.500
    w133 - 0                                     17.210
    w144 - 0                                    199.838
    w155 - 0                                    198.322
    w211 - 0                                      8.377
    w222 - 0                                      3.260
    w233 - 0                                      9.067
    w244 - 0                                    347.816
    w255 - 0                                     38.493

>
> # The estimate of psi22 is a bit below zero. The search got stuck on the boundary
instead of bouncing off.
> # This search was unsuccessful. We need better starting values.
>
```

```
> # Now obtain the MOM estimates to use as starting values.
> # Using explicit solutions from the textbook ...


> head(bmidata)
  age1 bmi1 fat1 cholest1 diastol1 age2 bmi2 fat2 cholest2 diastol2
1   63 24.5 16.5    195.4       38   60 23.9 20.1    203.5       66
2   42 13.0  1.9    184.3       86   44 14.8  2.6    197.3       78
3   32 22.5 14.6    354.1      104   33 21.7 20.4    374.3       73
4   59 25.5 19.0    214.6       93   58 28.5 20.0    203.7      106
5   45 26.5 17.8    324.8       97   43 25.0 12.3    329.7       92
6   31 19.4 17.1    280.7       92   42 19.9 19.9    276.7       87

> W1 = as.matrix(bmidata[,1:3])  # age1 bmi1 fat1
> V1 = as.matrix(bmidata[,4:5])  # cholest1 diastol1
> W2 = as.matrix(bmidata[,6:8])  # age2 bmi2 fat2
> V2 = as.matrix(bmidata[,9:10]) # cholest2 diastol2
> var(W1,W2) # Matrix of sample covariances
           age2      bmi2     fat2
age1 148.220782  3.621581 25.29808
bmi1   5.035726 13.194016 21.42201
fat1  23.542289 20.613490 45.13296
> # Using S as short for Sigmahat, and not worrying about n vs. n-1,
> S11 = var(W1); S12 = var(W1,V1); S13 = var(W1,W2); S14 = var(W1,V2)
>                S22 = var(V1);    S23 = var(V1,W2); S24 = var(V1,V2)
>                                  S33 = var(W2);    S34 = var(W2,V2)
>                                                    S44 = var(V2)
> # The matrices below should all have "hat" in the name, because they are
estimates
> Phi = (S13+t(S13))/2
> rownames(Phi) = colnames(Phi) = c('age','bmi','fat'); Phi
           age       bmi      fat
age 148.220782  4.328654 24.42019
bmi   4.328654 13.194016 21.01775
fat  24.420185 21.017749 45.13296
> # To my surprise, these are quite close to the MLEs from the first run.
> Beta = 0.5*(t(S14)+S23) %*% solve(Phi)
> rownames(Beta) = c('cholest','diastol')
> colnames(Beta) = c('age','bmi','fat'); Beta
               age        bmi      fat
cholest -0.3851327 -0.1885072 2.968322
diastol  0.0224190 -0.3556138 1.407425
> # These are miles away from the supposed MLEs
> # Can just say some of the rest are close and others are not.
>
> Psi = S24 - Beta %*% Phi %*% t(Beta)
> rownames(Psi) = colnames(Psi) = c('cholest','diastol') # epsilon1, epsilon2
> Psi
           cholest   diastol
cholest 2548.17303 -44.56069
diastol  -28.70087  57.64153
> # Oops, it should be symmetric.
> Psi = ( Psi+t(Psi) )/2; Psi
           cholest   diastol
cholest 2548.17303 -36.63078
diastol  -36.63078  57.64153
> # Again, far away.
> Omega11 = S11 - Phi; Omega11
          age1     bmi1      fat1
age1 19.640040 4.610807  1.634183
bmi1  4.610807 8.699533  8.754484
fat1  1.634183 8.754484 15.033932
> # Supposed MLEs are pretty close here.
```

```
> Omega12 = S12 - ( S14+t(S23) )/2; Omega12 # Not too bad
      cholest1  diastol1
age1  4.499017 12.164192
bmi1 -1.517733 10.671443
fat1  3.888565 -2.196681
> Omega22 = S22-S24 # A little rough but consistent
> Omega22 = (Omega22 + t(Omega22)  )/2
> Omega22 # Variances okay, covariance off.
          cholest1  diastol1
cholest1 213.76117  11.24971
diastol1  11.24971 196.44520
> Omega33 = S33 - Phi; Omega33 # Not too bad
          age2      bmi2      fat2
age2  5.862661 -1.219843 -2.155736
bmi2 -1.219843  1.146991 -1.714769
fat2 -2.155736 -1.714769 10.033984
> Omega34 = S34 - ( S14+t(S23) )/2; Omega34 # Not too bad
      cholest2   diastol2
age2 -2.978041  0.7795992
bmi2 -1.206256  2.1081739
fat2 -6.422983 -4.9125882
> Omega44 = S44 - S24 ; Omega44 = ( Omega44 + t(Omega44) )/2
> Omega44 # Not terrible
          cholest2  diastol2
cholest2 333.45335 -21.65923
diastol2 -21.65923  47.23065
>
> round(Beta,3)
           age    bmi   fat
cholest -0.385 -0.189 2.968
diastol  0.022 -0.356 1.407
>
> # Unfortunately this model is going to be long. Specifying a full set of starting
values is a pain in the ass. Use           start()
>
> bmimodel2 =
+     ############################################################
+     # Latent variable model
+     # ---------------------
+     'cholest ~ beta11*age        + beta12*bmi        + beta13*fat +
+                 start(-0.385)*age + start(-0.189)*bmi + start(2.968)*fat
+     diastol ~ beta21*age         + beta22*bmi        + beta23*fat +
+                start(0.022)*age   + start(-0.356)*bmi + start(1.407)*fat
+     #
+     # Measurement model
+     # -----------------
+     age =~ 1*age1 + 1*age2
+     bmi =~ 1*bmi1 + 1*bmi2
+     fat =~ 1*fat1 +1*fat2
+     cholest =~ 1*cholest1 + 1*cholest2
+     diastol =~ 1*diastol1 + 1*diastol2
+     #
+     # Variances and covariances
+     # -------------------------
+     # Of latent explanatory variables
+         age ~~ phi11*age + start(148.220782)*age
+         age ~~ phi12*bmi + start(4.328654)*bmi
+         age ~~ phi13*fat + start(24.42019)*fat
+         bmi ~~ phi22*bmi + start(13.194016)*bmi
+         bmi ~~ phi23*fat + start(21.01775)*fat
+         fat ~~ phi33*fat + start(45.13296)*fat
+     # Of error terms in latent the regression (epsilon_ij)
+         cholest ~~ psi11*cholest + start(2548.17303)*cholest
+         cholest ~~ psi12*diastol + start(-36.63078)*diastol
+         diastol ~~ psi22*diastol + start(57.64153)*diastol
```

```
+     # Of measurement errors (e_ijk) for measurement set 1
+     age1 ~~ w111*age1; age1 ~~ w112*bmi1; age1 ~~ w113*fat1;
+     age1 ~~ w114*cholest1; age1 ~~ w115*diastol1
+             bmi1 ~~ w122*bmi1; bmi1 ~~ w123*fat1; bmi1 ~~ w124*cholest1; bmi1 ~~ w125*diastol1
+                     fat1 ~~ w133*fat1; fat1 ~~ w134*cholest1; fat1 ~~ w135*diastol1
+                             cholest1 ~~ w144*cholest1; cholest1 ~~ w145*diastol1
+                                                        diastol1 ~~ w155*diastol1
+     # Of measurement errors (e_ijk) for measurement set 2
+     age2 ~~ w211*age2; age2 ~~ w212*bmi2; age2 ~~ w213*fat2;
+     age2 ~~ w214*cholest2; age2 ~~ w215*diastol2
+             bmi2 ~~ w222*bmi2; bmi2 ~~ w223*fat2; bmi2 ~~ w224*cholest2; bmi2 ~~ w225*diastol2
+                     fat2 ~~ w233*fat2; fat2 ~~ w234*cholest2; fat2 ~~ w235*diastol2
+                             cholest2 ~~ w244*cholest2; cholest2 ~~ w245*diastol2
+                                                        diastol2 ~~ w255*diastol2
+      # Bounds (Variances are positive)
+      # ------
+      phi11 > 0; phi22 > 0 ; phi33 > 0
+      psi11 > 0; psi22 > 0
+      w111 > 0; w122 > 0; w133 > 0; w144 > 0; w155 > 0;
+      w211 > 0; w222 > 0; w233 > 0; w244 > 0; w255 > 0
+      ' ################## End of bmimodel1 ##################
> fit2 = lavaan(bmimodel2, data=bmidata)
>
> show(fit2)
lavaan 0.6-3 ended normally after 495 iterations

  Optimization method                           NLMINB
  Number of free parameters                         45
  Number of inequality constraints                  15

  Number of observations                           500

  Estimator                                         ML
  Model Fit Test Statistic                       4.654
  Degrees of freedom                                10
  P-value (Chi-square)                           0.913
>
> # Exactly on the money; Chi-squared = 4.6537.
>
> summary(fit2)
lavaan 0.6-3 ended normally after 495 iterations

  Optimization method                           NLMINB
  Number of free parameters                         45
  Number of inequality constraints                  15

  Number of observations                           500

  Estimator                                         ML
  Model Fit Test Statistic                       4.654
  Degrees of freedom                                10
  P-value (Chi-square)                           0.913

Parameter Estimates:

  Information                                 Expected
  Information saturated (h1) model          Structured
  Standard Errors                             Standard
```

Latent Variables:

		Estimate	Std.Err	z-value	P(>|z|)
age =~					
age1		1.000			
age2		1.000			
bmi =~					
bmi1		1.000			
bmi2		1.000			
fat =~					
fat1		1.000			
fat2		1.000			
cholest =~					
cholest1		1.000			
cholest2		1.000			
diastol =~					
diastol1		1.000			
diastol2		1.000			

Regressions:

		Estimate	Std.Err	z-value	P(>|z|)
cholest ~					
age	(bt11)	-0.320	0.228	-1.404	0.160
bmi	(bt12)	0.393	1.708	0.230	0.818
fat	(bt13)	2.774	0.980	2.830	0.005
diastol ~					
age	(bt21)	0.020	0.050	0.407	0.684
bmi	(bt22)	-0.480	0.419	-1.145	0.252
fat	(bt23)	1.480	0.235	6.312	0.000

Covariances:

		Estimate	Std.Err	z-value	P(>|z|)
age ~~					
bmi	(ph12)	4.160	2.141	1.944	0.052
fat	(ph13)	23.320	3.986	5.851	0.000
bmi ~~					
fat	(ph23)	20.976	1.584	13.244	0.000
.cholest ~~					
.diastol	(ps12)	-45.869	24.969	-1.837	0.066
.age1 ~~					
.bmi1	(w112)	3.998	0.945	4.231	0.000
.fat1	(w113)	2.389	1.505	1.587	0.113
.cholst1	(w114)	2.694	9.091	0.296	0.767
.diastl1	(w115)	10.562	3.824	2.762	0.006
.bmi1 ~~					
.fat1	(w123)	8.969	0.956	9.382	0.000
.cholst1	(w124)	-0.884	4.178	-0.212	0.832
.diastl1	(w125)	10.059	2.274	4.423	0.000
.fat1 ~~					
.cholst1	(w134)	7.920	6.741	1.175	0.240
.diastl1	(w135)	-2.930	3.409	-0.859	0.390
.cholest1 ~~					
.diastl1	(w145)	-0.094	16.907	-0.006	0.996
.age2 ~~					
.bmi2	(w212)	-0.660	0.735	-0.899	0.369
.fat2	(w213)	-2.703	1.369	-1.974	0.048
.cholst2	(w214)	-1.953	8.962	-0.218	0.827
.diastl2	(w215)	2.274	2.710	0.839	0.401
.bmi2 ~~					
.fat2	(w223)	-1.850	0.705	-2.624	0.009
.cholst2	(w224)	-2.652	3.476	-0.763	0.446
.diastl2	(w225)	2.651	1.487	1.784	0.074
.fat2 ~~					
.cholst2	(w234)	-11.373	6.546	-1.737	0.082
.diastl2	(w235)	-4.842	2.536	-1.909	0.056

```
 .cholest2 ~~
   .diastl2 (w245)   -8.969   12.605   -0.712    0.477

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)
    age     (ph11)  147.330    9.699   15.190    0.000
    bmi     (ph22)   13.341    0.986   13.528    0.000
    fat     (ph33)   44.485    3.101   14.345    0.000
   .cholest (ps11) 2534.496  171.258   14.799    0.000
   .diastol (ps22)   56.165    9.221    6.091    0.000
   .age1    (w111)   18.584    2.914    6.378    0.000
   .bmi1    (w122)    8.665    0.708   12.239    0.000
   .fat1    (w133)   16.125    1.659    9.717    0.000
   .cholst1 (w144)  200.105   57.422    3.485    0.000
   .diastl1 (w155)  195.042   14.323   13.617    0.000
   .age2    (w211)    6.861    2.701    2.540    0.011
   .bmi2    (w222)    1.089    0.491    2.220    0.026
   .fat2    (w233)    9.331    1.539    6.064    0.000
   .cholst2 (w244)  344.453   60.290    5.713    0.000
   .diastl2 (w255)   48.350    8.246    5.864    0.000

Constraints:
                                                |Slack|
    phi11 - 0                                   147.330
    phi22 - 0                                    13.341
    phi33 - 0                                    44.485
    psi11 - 0                                  2534.496
    psi22 - 0                                    56.165
    w111 - 0                                     18.584
    w122 - 0                                      8.665
    w133 - 0                                     16.125
    w144 - 0                                    200.105
    w155 - 0                                    195.042
    w211 - 0                                      6.861
    w222 - 0                                      1.089
    w233 - 0                                      9.331
    w244 - 0                                    344.453
    w255 - 0                                     48.350

>
> # It took 495 iterations with these very good starting values. SAS took 5 with
its automatic starting values. Of course I did not specify ALL the starting values.
There were 30 more. I guess I better do it, to see how much faster it is with all
the starting values.
```

```
>
> bmimodel2 =
+     #
+     # Latent variable model
+     # ---------------------
+         'cholest ~ beta11*age        + beta12*bmi        + beta13*fat +
+                    start(-0.385)*age + start(-0.189)*bmi + start(2.968)*fat
+          diastol ~ beta21*age        + beta22*bmi        + beta23*fat +
+                    start(0.022)*age  + start(-0.356)*bmi + start(1.407)*fat
+     #
+     # Measurement model
+     # -----------------
+         age =~ 1*age1 + 1*age2
+         bmi =~ 1*bmi1 + 1*bmi2
+         fat =~ 1*fat1 +1*fat2
+         cholest =~ 1*cholest1 + 1*cholest2
+         diastol =~ 1*diastol1 + 1*diastol2
+     #
+     # Variances and covariances
+     # -------------------------
+     # Of latent explanatory variables
+         age ~~ phi11*age + start(148.220782)*age
+         age ~~ phi12*bmi + start(4.328654)*bmi
+         age ~~ phi13*fat + start(24.42019)*fat
+         bmi ~~ phi22*bmi + start(13.194016)*bmi
+         bmi ~~ phi23*fat + start(21.01775)*fat
+         fat ~~ phi33*fat + start(45.13296)*fat
+     # Of error terms in latent the regression (epsilon_ij)
+         cholest ~~ psi11*cholest + start(2548.17303)*cholest
+         cholest ~~ psi12*diastol + start(-36.63078)*diastol
+         diastol ~~ psi22*diastol + start(57.64153)*diastol
+     # Of measurement errors (e_ijk) for measurement set 1
+         age1 ~~ w111*age1 + start(19.640040)*age1
+         age1 ~~ w112*bmi1 + start(4.610807)*bmi1
+         age1 ~~ w113*fat1 + start(1.634183)*fat1
+         age1 ~~ w114*cholest1 + start(4.499017)*cholest1
+         age1 ~~ w115*diastol1 + start(12.164192)*diastol1
+         bmi1 ~~ w122*bmi1 + start(8.699533)*bmi1
+         bmi1 ~~ w123*fat1 + start(8.754484)*fat1
+         bmi1 ~~ w124*cholest1 + start(-1.517733)*cholest1
+         bmi1 ~~ w125*diastol1 + start(10.671443)*diastol1
+         fat1 ~~ w133*fat1 + start(15.033932)*fat1
+         fat1 ~~ w134*cholest1 + start(3.888565)*cholest1
+         fat1 ~~ w135*diastol1 + start(-2.196681)*diastol1
+         cholest1 ~~ w144*cholest1 + start(213.76117)*cholest1
+         cholest1 ~~ w145*diastol1 + start(11.24971)*diastol1
+         diastol1 ~~ w155*diastol1 + start(196.44520)*diastol1
+     # Of measurement errors (e_ijk) for measurement set 2
+         age2 ~~ w211*age2 + start(5.862661)*age2
+         age2 ~~ w212*bmi2 + start(-1.219843)*bmi2
+         age2 ~~ w213*fat2 + start(-2.155736)*fat2
+         age2 ~~ w214*cholest2 + start(-2.978041)*cholest2
+         age2 ~~ w215*diastol2 + start(0.7795992)*diastol2
+         bmi2 ~~ w222*bmi2 + start(1.146991)*bmi2
+         bmi2 ~~ w223*fat2 + start(-1.714769)*fat2
+         bmi2 ~~ w224*cholest2 + start(-1.206256)*cholest2
+         bmi2 ~~ w225*diastol2 + start(2.1081739)*diastol2
+         fat2 ~~ w233*fat2 + start(10.033984)*fat2
+         fat2 ~~ w234*cholest2 + start(-6.422983)*cholest2
+         fat2 ~~ w235*diastol2 + start(-4.9125882)*diastol2
+         cholest2 ~~ w244*cholest2 + start(333.45335)*cholest2
+         cholest2 ~~ w245*diastol2 + start(-21.65923)*diastol2
+         diastol2 ~~ w255*diastol2 + start(47.23065)*diastol2
```

```
+     # Bounds (Variances are positive)
+     # ------
+         phi11 > 0; phi22 > 0 ; phi33 > 0
+         psi11 > 0; psi22 > 0
+         w111 > 0; w122 > 0; w133 > 0; w144 > 0; w155 > 0;
+         w211 > 0; w222 > 0; w233 > 0; w244 > 0; w255 > 0
+     ' ################# End of bmimodel2 #################
>
> fit2 = lavaan(bmimodel2, data=bmidata)
> show(fit2) # 327 iterations, on the money.
lavaan 0.6-3 ended normally after 327 iterations

  Optimization method                           NLMINB
  Number of free parameters                         45
  Number of inequality constraints                  15

  Number of observations                           500

  Estimator                                         ML
  Model Fit Test Statistic                       4.654
  Degrees of freedom                                10
  P-value (Chi-square)                           0.913
>
> # Now a LR test of BMI, H0: beta12 = beta22 = 0
> # Will the constraint conflict with the starting values?
>
> nobmi = lavaan(bmimodel2, data=bmidata,
+             constraints = 'beta12 == 0
+                            beta22 == 0')
>
> anova(nobmi,fit2)
Chi Square Difference Test

      Df   AIC   BIC  Chisq Chisq diff Df diff Pr(>Chisq)
fit2  10 35758 35947 4.6537
nobmi 12 35755 35936 6.1457      1.492       2     0.4743
>
```
