Cumulative hazard transformation

- We begin with the following useful theorem:
- **Theorem:** Suppose T is a continuous nonnegative random variable with cumulative hazard function Λ. Then the random variable $Y = \Lambda(T)$ follows an exponential distribution with rate $\lambda = 1$.
- Thus, one way of checking the validity of a model is by comparing the model's estimates $\{\hat{\Lambda}(t_i)\}$ against the standard exponential distribution

Cumulative Hazard Transformation

$T \sim f_T(t) \quad P(T>0)=1 \qquad H(t) = \int_0^t h(x)\,dx$, but use

$$S(t) = e^{-H(t)} \iff -\log S(t) = H(t)$$

Let $Y = H(T) \quad P(Y>0)=1$

For $y>0$

$$F_Y(y) = P(Y \le y) = P(H(T) \le y)$$

$$= P(-\log S(T) \le y) = P(\log S(T) \ge -y)$$

$$= P(S(T) \ge e^{-y}) = P(1 - F_T(T) \ge e^{-y})$$

$$= P(F_T(T) \le 1 - e^{-y}) = P(F_T^{-1}(F_T(T)) \le F_T^{-1}(1-e^{-y}))$$

$$= P(T \le F_T^{-1}(1-e^{-y})) = F_T(F_T^{-1}(1-e^{-y}))$$

$$= 1 - e^{-y}$$ CDF of Exponential with $\lambda = 1$

$$= F_Y(y)$$

for $y>0$