Sample Questions: Survival and Hazard Functions

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For all these questions, T is a continuous random variable with $P(T > 0) = 1$, density $f(t)$ and cumulatve distribution function $F(t) = P(T \leq t)$.

1. The survival function is $S(t) = P(T > t)$. Prove $E(T) = \int_0^\infty S(t)\, dt$.

2. The hazard function is defined by $h(t) = \lim_{\Delta \to 0} \frac{P(t < T < t + \Delta | T > t)}{\Delta}$, where $\Delta > 0$. Prove $h(t) = \frac{f(t)}{S(t)}$.

3. Prove $S(t) = e^{-\int_0^t h(x)\,dx}$.

4. Let $T \sim \exp(\lambda)$. Find the hazard function $h(t)$ for $t > 0$.

5. Let T have the Pareto density $f(t|\theta) = \begin{cases} \frac{\theta}{t^{\theta+1}} & \text{for } t \geq 1 \\ 0 & \text{for } t < 1 \end{cases}$

   (a) Find the hazard function $h(t)$ for $t > 1$.

(b) Earlier, we found the MLE $\widehat{\theta}_n = \frac{n}{\sum_{i=1}^{n} \log t_i}$, and $\widehat{v}_n = \frac{\widehat{\theta}_n^2}{n}$.

i. Give $\widehat{h(t)}$, the maximum likelihood estimate of the hazard function evaluated at a particular time $t > 1$. Your answer is a formula involving t and $\widehat{\theta}_n$.

ii. We want a confidence interval for $h(t)$, the hazard function evaluated at a particular time $t > 1$. Give formulas for the lower and upper 95% confidence limits. Show your work.

6. Let T have a gamma distribution with parameters $\alpha > 0$ and $\lambda > 0$.

   (a) What is the hazard function?

   (b) Using R, plot the hazard function for several values of α and λ. How do the parameter values influence the shape of the hazard function?
