

# Sample Questions: Survival and Hazard Functions

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For all these questions,  $T$  is a continuous random variable with  $P(T > 0) = 1$ , density  $f(t)$  and cumulative distribution function  $F(t) = P(T \leq t)$ .

1. The survival function is  $S(t) = P(T > t)$ . Prove  $E(T) = \int_0^{\infty} S(t) dt$ .

2. The hazard function is defined by  $h(t) = \lim_{\Delta \rightarrow 0} \frac{P(t < T < t + \Delta | T > t)}{\Delta}$ ,  
where  $\Delta > 0$ . Prove  $h(t) = \frac{f(t)}{S(t)}$ .

3. Prove  $S(t) = e^{-\int_0^t h(x) dx}$ .

4. Let  $T \sim \exp(\lambda)$ . Find the hazard function  $h(t)$  for  $t > 0$ .

5. Let  $T$  have the Pareto density  $f(t|\theta) = \begin{cases} \frac{\theta}{t^{\theta+1}} & \text{for } t \geq 1 \\ 0 & \text{for } t < 1 \end{cases}$

(a) Find the hazard function  $h(t)$  for  $t > 1$ .

(b) Earlier, we found the MLE  $\hat{\theta}_n = \frac{n}{\sum_{i=1}^n \log t_i}$ , and  $\hat{v}_n = \frac{\hat{\theta}_n^2}{n}$ .

i. Give  $\widehat{h}(t)$ , the maximum likelihood estimate of the hazard function evaluated at a particular time  $t > 1$ . Your answer is a formula involving  $t$  and  $\hat{\theta}_n$ .

ii. We want a confidence interval for  $h(t)$ , the hazard function evaluated at a particular time  $t > 1$ . Give formulas for the lower and upper 95% confidence limits. Show your work.

6. Let  $T$  have a gamma distribution with parameters  $\alpha > 0$  and  $\lambda > 0$ .

(a) What is the hazard function?

(b) Using R, plot the hazard function for several values of  $\alpha$  and  $\lambda$ . How do the parameter values influence the shape of the hazard function?

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<http://www.utstat.toronto.edu/brunner/oldclass/312f23>