Name Jerry

Student Number _____

STA 442/2101 f2014 Quiz 4

1. The last question of Assignment 4 (Problem 19) is based upon the Distraction data. In part (d) of Problem 13, you were asked to test the difference between the average expected test performance for Voice distraction and the average expected test performance for Music distraction.

   (a) (1 point) What is the value of the test statistic W_n? The answer is a number from your printout. **Write the number from your printout in the space below. On your printout, circle the value of W_n and write "Question 1a" beside it.**

   $$W_n = 567.3154$$

   (b) (1 point) What is the p-value? The answer is a number from your printout. **Write the number from your printout in the space below. On your printout, circle the p-value and write "Question 1b" beside it.**

   $$p = 0.0000$$

   (c) (2 points) In plain, non-statistical language, what do you conclude from this test? ~~The answer is a short statement about average expected performance.~~
   For simplicity, please refer to average expected performance as "average performance."

   Average performance is worse with Voice distraction.

   (or better with Music distraction)

   −1 for statement of difference in performance without saying what the difference is.

2. Let $Z_1, \ldots, Z_p \overset{i.i.d.}{\sim} N(0,1)$, let $\mathbf{Z} = (Z_1, \ldots, Z_p)^\top$, and let $\mathbf{Y} = \Sigma^{1/2}\mathbf{Z} + \boldsymbol{\mu}$, where Σ is a $p \times p$ symmetric *non-negative definite* matrix and $\boldsymbol{\mu} \in \mathbb{R}^p$.

(a) (1 point) What is $E(\mathbf{Y})$? You do *not* need to show any work.

$$E(Y) = \mu$$

(b) (2 points) What is the variance-covariance matrix of $\mathbf{Y}$? Show some work.

$$V(Y) = V(\Sigma^{1/2} Z) = \Sigma^{1/2} I \Sigma^{1/2} = \Sigma$$

(c) (3 points) What is the moment-generating function of $\mathbf{Y}$? Show your work. Use the formula sheet. Start with the moment-generating function

$$M_Z(\underset{\sim}{t}) = \prod_{i=1}^{n} M_{Z_i}(t_i) = \prod_{i=1}^{n} e^{\frac{1}{2}t_i^2} = e^{\frac{1}{2}\Sigma t_i^2} = e^{\frac{1}{2}t^\top t},$$

So $M_Y(t) = M_{\Sigma^{1/2}Z+\mu}(t) = e^{t^\top \mu} M_Z(\Sigma^{1/2^\top} t)$

$$= e^{t^\top \mu} e^{\frac{1}{2}(\Sigma^{1/2}t)^\top \Sigma^{1/2}t} = e^{t^\top \mu} e^{\frac{1}{2}t^\top \Sigma^{1/2}\Sigma^{1/2}t}$$

$$= e^{t^\top \mu + \frac{1}{2}t^\top \Sigma t}$$

Attach your R printout to the quiz. Make sure your name and student number are on your printout.