Name Jerry

Student Number _____

# STA 442/2101 f2014 Quiz 3

1. (3 points) In this simple regression through the origin, values of the explanatory variable are random, not fixed constants. Independently for $i = 1, \ldots, n$, let $Y_i = \beta X_i + \epsilon_i$, where $E(X_i) = E(\epsilon_i) = 0$, $Var(X_i) = \sigma_x^2$, $Var(\epsilon_i) = \sigma^2$, and $\epsilon_i$ is independent of $X_i$. Let $\widehat{\beta}_n = \frac{\sum_{i=1}^n X_i Y_i}{\sum_{i=1}^n X_i^2}$. Is $\widehat{\beta}_n$ a consistent estimator of $\beta$? Answer Yes or No and prove your answer.

$E(X_i Y_i) = E\{X_i(\beta X_i + \varepsilon_i)\}$

$= \beta E(X_i^2) + E(X_i \varepsilon_i) = \beta \sigma_x^2 + E(X_i)E(\varepsilon_i)$

$= \beta \sigma_x^2 + 0$

And $E(X_i^2) = \sigma_x^2 + 0^2 = \sigma_x^2$,

So $\frac{1}{n}\sum_{i=1}^n X_i Y_i \xrightarrow{as} \beta\sigma_x^2$ by LLN and

$\frac{1}{n}\sum_{i=1}^n X_i^2 \xrightarrow{as} \sigma_x^2$ by LLN, and

by continuous mapping,

$$\hat{\beta}_n = \frac{\frac{1}{n}\sum_{i=1}^n X_i Y_i}{\frac{1}{n}\sum_{i=1}^n X_i^2} \xrightarrow{as} \frac{\beta\sigma_x^2}{\sigma_x^2} = \beta$$

Consistent, (YES) (strongly consistent)

2. The last question of Assignment 3 (Problem 19) is based the model $X_1, \ldots, X_n \overset{i.i.d.}{\sim} N(\mu, \sigma^2)$.

   (a) (2 points) In part (d) of Problem 19, you were asked to calculate the $p$-value for a test about $\sigma^2$. **Write the $p$-value from your printout in the space below. On your printout, circle the $p$-value and write "Question 2a" beside it.**

$p = 0.0164$

   (b) (2 points) Do you stop the assembly line based on the $p$-value? **Answer Yes or No**. Use $\alpha = 0.05$. You can get credit for this part only if the $p$-value is correct.

YES

   (c) (3 points) In the second part of (i) in Problem 19, you were asked to calculate power of the test for $n = 10$ when the true value of $\sigma$ was equal to four. **Write the power value from your printout (a number between zero and one) in the space below. On your printout, circle the power value and write "Question 2c" beside it.**

Power = 0.391

**Attach your R printout to the quiz. Make sure your name and student number are on your printout.**