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Name Jerry

Student Number

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# STA 442/2101 f2014 Quiz 1

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1. (5 points) A random sample of size $n$ was drawn from a distribution with density $f(y) = \theta e^{-\theta y}$ for $y > 0$, where the parameter $\theta > 0$.

   (a) Find the maximum likelihood estimator of $\theta$. Show your work. Don't bother wth a second derivative test. Your answer is a symbolic expression. **Circle your final answer.**

$$\ell(\theta) = \log \prod_{i=1}^{n} \theta e^{-\theta y_i} = \log\left(\theta^n e^{-\theta \sum y_i}\right)$$

$$= n \log \theta - \theta \sum_{i=1}^{n} y_i$$

$$\frac{d\ell}{d\theta} = \frac{n}{\theta} - \sum y_i \overset{set}{=} 0 \Rightarrow \frac{n}{\theta} = \sum y_i$$

$$\Rightarrow \boxed{\theta = \frac{n}{\sum_{i=1}^{n} y_i} = \hat{\theta}}$$

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   (b) The data values are 4.1, 9.3, 2.2, 4.4. Give the maximum likelihood estimate in numeric form. Your answer is a number. **Circle it.**

$$\hat{\theta} = \frac{4}{20} = 1/5$$

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2. (5 points) Let $\mathbf{X}$ be a real $n \times p$ matrix, and let $\mathbf{a}$ be a real $p \times 1$ column vector. Show that $\mathbf{a}'(\mathbf{X}'\mathbf{X})\mathbf{a} \geq 0$ (that is, $\mathbf{X}'\mathbf{X}$ is non-negative definite). You have a lot more room than you need.

$$a'(x'x)a = (a'x')\,xa$$

$$= \underbrace{(xa)'}_{\in \mathbb{R}^{n\times 1}} \underbrace{xa}_{\in \mathbb{R}^{n\times 1}} = \sum_{j=1}^{n} z_j^2 \geq 0$$